Challenge Exercise 4
MATH 2275 – Winter 2006
Due Date: March 15, 2006

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the solutions correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name. For those of you in Math 2231/2233 (Ring/Group Theory), you are encouraged to write you solutions as a formal proof.

Problem. Let $\mathcal{M}_{m,n}$ denote the set of all $m \times n$ matrices with entries in $\mathbb{R}$, i.e.,

$$\mathcal{M}_{m,n} = \{M \mid M \text{ is a } m \times n \text{ matrix}\}$$

(*a*) [**3pts**] Show that $\mathcal{M}_{m,n}$ is vector space (see Definition on page 217).

(*b*) [**3pts**] Let $A, B \in \mathcal{M}_{m,n}$. Show that the trace (see Exercise 25 on page 334) of the matrix $B^T A$ is given by the formula

$$\operatorname{tr}(B^T A) = \sum_{i=1}^{n} \sum_{k=1}^{m} b_{ki} a_{ki}$$

where $A = [a_{ij}]$ and $B = [b_{ij}]$.

(*c*) [**4pts**] Show that the function

$$\langle A, B \rangle = \operatorname{tr}(B^t A)$$

is an inner product on the vector space $\mathcal{M}_{m,n}$.