Challenge Exercise 5
MATH 2275 – Winter 2006
Due Date: March 29, 2006

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the solutions correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name. For those of you in Math 2231/2233 (Ring/Group Theory), you are encouraged to write you solutions as a formal proof.

Problem. We begin with a definition:

Definition. An $n \times n$ matrix A is a *skew-symmetric* matrix if $A = -A^T$.

We look at some properties of a skew-symmetric matrix.

(*a*) [**2pts**] Find an example of 3×3 skew-symmetric matrix.

(*b*) [**2pts**] Prove that the diagonal entries of any skew-symmetric matrix A must all be 0.

(*c*) [**2pts**] If A_1 and A_2 are skew-symmetric matrices, are $A_1 + A_2$ and $A_1 A_2$ also skew-symmetric?

(*d*) [**2pts**] Let A be a 2×2 skew-symmetric matrix. Show that each non-zero eigenvalue of A is a pure imaginary number, i.e. $\lambda = ci$ for some $c \in \mathbb{R}$.

(*e*) [**2pts**] Show that the eigenvectors associated with distinct eigenvalues of a skew-symmetric 2×2 matrix must be orthogonal.