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## SEMINAR INFORMATION (Winter 2006)

### Combinatorial Commutative Algebra

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The aim of this seminar is to introduce the basic objects of combinatorial commutative algebra. We will meet once a week to talk about some results in this field. Whenever possible, we will work through the original research articles.

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Time Thursdays (time to be set)  
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**References.** Instead of using only one textbook, we will make use of a wide variety of sources for this class, and in many cases, research articles. Below is our primary list of references:

*Books:*

- (1) W. Bruns, J. Herzog, *Cohen-Macaulay rings*. Cambridge, 1993.
- (2) D. Cox, J. Little, D. O'Shea, *Using Algebraic Geometry*. Springer, 1998.
- (3) E. Miller, B. Sturmfels, *Combinatorial Commutative Algebra*. Springer, 2004.
- (4) R. Stanley, *Combinatorics and commutative algebra*. Birkhaeuser 1983.
- (5) R. Villarreal, *Monomial Algebras*. Marcel Dekker, 2001.

*Research Articles:*

- (1) J. Eagon, V. Reiner, Resolutions of Stanley-Reisner rings and Alexander duality. *Journal of Pure and Applied Algebra* **130** (1998) 265-275.
- (2) P. Frankl, A new short proof for the Kruskal-Katona Theorem. *Discrete Mathematics* **48** (1984) 327-329.
- (3) S. Faridi, The facet ideal of a simplicial complex. *Manuscripta Mathematica* **109** (2005) 159-174.
- (4) H.T. Hà, A. Van Tuyl, Splittable ideals and the resolutions of monomial ideals. Preprint (2005)
- (5) S. Jacques, M. Katzman, The Betti numbers of forests. Preprint (2005)

**Outline.** A separate handout describing the topics is attached.

**Course Requirements (Evaluation).** You will be required to do two things: (1) you will required to give two lectures, and (2) some homework problems will be provided for you to work on. Each component will be worth 50%. We will meet once a week (time and place to be determined) for an hour to an hour and half to discuss a new topic.

Another skill I want you to learn is how to typeset mathematics. I will explain how to use L<sup>A</sup>T<sub>E</sub>X, and I want you to type up the lectures given in the seminar.

## Combinatorial Commutative Algebra Seminar Schedule

Lecture 1	<i>Title:</i>	<b>Introduction to Simplicial Complexes</b>
	<i>Topics:</i>	Basic definitions and examples of simplicial complexes
	<i>References:</i>	Burns & Herzog, Miller & Sturmfels, Stanley
Lecture 2	<i>Title:</i>	<b>Kruskal-Katona's Theorem</b>
	<i>Topics:</i>	$f$ -vectors and their classification
	<i>References:</i>	Frankl
Lecture 3	<i>Title:</i>	<b>Stanley-Reisner Correspondence</b>
	<i>Topics:</i>	Stanley-Reisner correspondence Hilbert functions and series $f$ -vectors and $h$ -vectors
	<i>References:</i>	Bruns & Herzog, Stanley
Lecture 4	<i>Title:</i>	<b>Cohen-Macaulay Rings I</b>
	<i>Topics:</i>	Definition of a Cohen-Macaulay rings Shellability
	<i>References:</i>	Bruns & Herzog
Lecture 5	<i>Title:</i>	<b>Cohen-Macaulay Rings II</b>
	<i>Topics:</i>	Continuation of Lecture 4
	<i>References:</i>	Bruns & Herzog
Lecture 6	<i>Title:</i>	<b>Edge and Facet Ideals</b>
	<i>Topics:</i>	Introduce facet ideals, an alternative correspondence
	<i>References:</i>	Faridi
Lecture 7	<i>Title:</i>	<b>Minimal Free Resolutions</b>
	<i>Topics:</i>	Minimal graded resolutions, graded Betti numbers Introduction to CoCoA, a computer program that computes Betti numbers
	<i>References:</i>	Cox, Little, & O'Shea
Lecture 8	<i>Title:</i>	<b>Reduced Simplicial Homology</b>
	<i>Topics:</i>	Brief introduction to reduced simplicial homology
	<i>References:</i>	(to be added)
Lecture 9	<i>Title:</i>	<b>Betti numbers of Stanley-Reisner rings</b>
	<i>Topics:</i>	Hochster's formula, Eagon-Reiner's formula
	<i>References:</i>	Eagon & Reiner
Lecture 10	<i>Title:</i>	<b>Splittable ideals and resolutions of trees</b>
	<i>Topics:</i>	splittable ideals, tress
	<i>References:</i>	Hà & Van Tuyl, Jacques & Katzman