
SEMINAR INFORMATION (Winter 2006)
Combinatorial Commutative Algebra

The aim of this seminar is to introduce the basic objects of combinatorial commutative algebra. We will meet once a week to talk about some results in this field. Whenever possible, we will work through the original research articles.

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References. Instead of using only one textbook, we will make use of a wide variety of sources for this class, and in many cases, research articles. Below is our primary list of references:

Books:

- (1) W. Bruns, J. Herzog, *Cohen-Macaulay rings*. Cambridge, 1993.
- (2) D. Cox, J. Little, D. O'Shea, *Using Algebraic Geometry*. Springer, 1998.
- (3) E. Miller, B. Sturmfels, *Combinatorial Commutative Algebra*. Springer, 2004.
- (4) R. Stanley, *Combinatorics and commutative algebra*. Birkhaeuser 1983.
- (5) R. Villarreal, *Monomial Algebras*. Marcel Dekker, 2001.

Research Articles:

- (1) J. Eagon, V. Reiner, Resolutions of Stanley-Reisner rings and Alexander duality. *Journal of Pure and Applied Algebra* **130** (1998) 265-275.
- (2) P. Frankl, A new short proof for the Kruskal-Katona Theorem. *Discrete Mathematics* **48** (1984) 327-329.
- (3) S. Faridi, The facet ideal of a simplicial complex. *Manuscripta Mathematica* **109** (2005) 159-174.
- (4) H.T. Hà, A. Van Tuyl, Splittable ideals and the resolutions of monomial ideals. Preprint (2005)
- (5) S. Jacques, M. Katzman, The Betti numbers of forests. Preprint (2005)

Outline. A separate handout describing the topics is attached.

Course Requirements (Evaluation). You will be required to do two things: (1) you will be required to give two lectures, and (2) some homework problems will be provided for you to work on. Each component will be worth 50%. We will meet once a week (time and place to be determined) for an hour to an hour and half to discuss a new topic.

Another skill I want you to learn is how to typeset mathematics. I will explain how to use \LaTeX , and I want you to type up the lectures given in the seminar.

Combinatorial Commutative Algebra Seminar Schedule

- Lecture 1 *Title:* **Introduction to Simplicial Complexes**
Topics: Basic definitions and examples of simplicial complexes
References: Burns & Herzog, Miller & Sturmfels, Stanley
- Lecture 2 *Title:* **Kruskal-Katona's Theorem**
Topics: f -vectors and their classification
References: Frankl
- Lecture 3 *Title:* **Stanley-Reisner Correspondence**
Topics: Stanley-Reisner correspondence
 Hilbert functions and series
 f -vectors and h -vectors
References: Bruns & Herzog, Stanley
- Lecture 4 *Title:* **Cohen-Macaulay Rings I**
Topics: Definition of a Cohen-Macaulay rings
 Shellability
References: Bruns & Herzog
- Lecture 5 *Title:* **Cohen-Macaulay Rings II**
Topics: Continuation of Lecture 4
References: Bruns & Herzog
- Lecture 6 *Title:* **Edge and Facet Ideals**
Topics: Introduce facet ideals, an alternative correspondence
References: Faridi
- Lecture 7 *Title:* **Minimal Free Resolutions**
Topics: Minimal graded resolutions, graded Betti numbers
 Introduction to CoCoA, a computer program that computes Betti numbers
References: Cox, Little, & O'Shea
- Lecture 8 *Title:* **Reduced Simplicial Homology**
Topics: Brief introduction to reduced simplicial homology
References: (to be added)
- Lecture 9 *Title:* **Betti numbers of Stanley-Reisner rings**
Topics: Hochster's formula, Eagon-Reiner's formula
References: Eagon & Reiner
- Lecture 10 *Title:* **Splittable ideals and resolutions of trees**
Topics: splittable ideals, tress
References: Hà & Van Tuyl, Jacques & Katzman