Challenge Exercise 2
MATH 1281 – 2007
Due Date: Nov. 9, 2007

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the problem correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name.

Problem 1. **[5pts]** Let A be any set, and let $f : A \to A$ be any function. We define a sequence of sets $\{A_n \mid n \in \mathbb{N}\}$ where

$$A_0 = A \text{ and } A_{n+1} = f(A_n) \text{ for all } n \geq 0.$$

Prove the following statements:

(a) For all $n \geq 0$, $A_{n+1} \subseteq A_n$.

(b) Let

$$A^* = \bigcap_{n \in \mathbb{N}} A_n.$$

Then $f(A^*) \subseteq A^*$.

Problem 2. **[5pts]** Let m and n be positive integers. Let f be the function from

$$X = \{0, 1, \ldots, m-1\}$$

to X, that is, $f : X \to X$, defined by

$$f(x) = nx \pmod{m}.$$

Prove that if f is one-to-one, then $gcd(m, n) = 1$. (Hint: Show the function f is also onto. Then there exists some $x \in X$ such that $f(x) = nx \equiv 1 \pmod{m}$.)