The purpose of this handout is to list the concepts, definitions, and results you will need to know for the final exam.

Final Exam Information. The final will be on

## Wednesday, Dec. 12, 2007 from 1PM-4PM in RB 2047

You will **not** be allowed to bring in any notes, use the text book, or use a calculator. **Bring your STUDENT ID**.

**Material Covered.** The exam will cover all the material discussed in class about Chapters 1 through 4. You will *not* be tested on MatLab. I have given a breakdown of what you will need to know from each section. For Chapters 1 and 2 please see your previous handout (copies can be found on the web).

**Section 3.1.** Know the definition of a determinant, and how to use Theorem 1 to compute the determinant of a matrix using the cofactor expansion down a row or column. Also know Theorem 2. Also know the trick for computing the determinant of a  $3 \times 3$  matrix (see page 191 before exercise 15).

**Section 3.2.** Know how a row operation changes the determinant of a matrix. Also know Theorems 4, 5, and 6.

**Section 3.3.** Know what Cramer's rule is, and how to use it to solve simple systems of linear equations. Know how to use this rule to find the inverse of a matrix. Also know Theorem 9, and know how it describes how area and volume change in a linear transformation.

**Section 4.1** Know the definition of a vector space, and know the two main examples of vector spaces,  $\mathbb{R}^n$  and  $\mathbb{P}_n$  (ignore Examples 3 and 5). Know the definition of a subspace, and you should know how to check whether a subset is a subspace. Also important is Theorem 1. Know how to do things like Example 11 (or Exercise 10).

Section 4.2 Know the definition of a null space of a matrix and the column space of a matrix. Know how to give an explicit description of Nul(A) (see Example 3). Understand the differences between these two spaces (see, for example, the table on page 232). Know what the kernel and range of a linear transformation are, and how they are related to Nul(A) and Col(A). Ignore Examples 8 and 9.

Section 4.3 Know what it means for a collection of elements in a vector space to be linearly independent and dependent. Know the definition of a basis. Also, know the Spanning Set Theorem. Be able to find the basis of Nul(A) and Col(A).

**Section 4.4** Know Theorem 7, and know what is meant by the  $\mathcal{B}$ -coordinate of a vector. Also know about the coordinate mapping. Also important is the notion of the change-of-coordinate matrix (see page 249). You can skip the material on isomorphisms (page 251-253).

**Section 4.5** Theorem 9 and 10 are very important, because they provide a justification for the definition of dimension. Know the definition of dimension. Know Theorem 11 and 12, and how to compute the dimensions of Nul(A) and Col(A).

Section 4.6 Know what the row space of a matrix is, and row to find a basis for Row(A) (see Theorem 13). Know the definition of the rank of a matrix, and know Theorem 14. You should be able to do problems like Example 3. Skip the subsection on applications to systems of equations, but know the connection between rank and the invertible matrix theorem.

**Section 4.7** Know what the change-of-coordinates matrix is, and know how to use Theorem 15. Be able to do problems like Example 2 and Example 3.

Section 4.8 You will not be tested on the material of this section.

**Section 4.9** Know the definition of a probability vector, stochastic matrix, and Markov chain. Know what is meant by a steady-state vector and how to find it (see Example 5). Understand Theorem 18.

**Note:** Sometime before the exam, I will try to calculate your mark up to the final. I will post the results on my website. Please double check your marks to make sure your assignment marks were recorded correctly.

I will be out of town Dec. 1-8. However, I should be in my office all day Dec. 10 and 11, and the morning of the exam to answer questions. Send me an email or call me (343-8228) to confirm. Good luck with your exams!—Adam

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