NAME: ________________________

STUDENT NUMBER: ________________________

MATH 1281 - Final Exam

Lakehead University

April 13, 2011

Dr. Adam Van Tuyl

Instructions: Answer all questions in the space provided. If you need more room, answer on the back of the page. Where appropriate, you must provide clear explanations.

You are *not* allowed to use a calculator. If a question involves a calculation you may leave it in an unexpanded form, e.g., you can write 5^4 instead of 625.

If doubt exists as to the interpretation of any question, you are urged to submit with the answer paper a clear statement of any assumptions made.

Page	Possible	Received
2	8	
3	8	
4	8	
5	6	
6	8	
7	8	
8	8	
9	8	
10	10	
11	10	
12	8	
13	10	
Total	100	

Chapter 1 – Logic and Proofs

1. **[4pts]** Find the truth table for the following compound proposition:

$$((P \to Q) \to (Q \to R)) \to (P \to R)$$

2. **[4pts]** Let n be any integer. Prove that if n^3 is odd, then n is odd. [Note: You may use any proof technique introduced in the course.]

Chapter 2 – Sets, Functions, Sequences, and Sums

3. **[4pts]** Let A and B be sets. Using a membership table, prove the following set identity

$$\overline{(\overline{A} \cap \overline{B})} = A \cup B.$$

4. **[4pts]** Consider the function $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(n) = 3\lceil \frac{n}{3} \rceil$. Is this function one-to-one? Is this function onto? Justify your answers.

Chapter 3 – Algorithms, Integers and Matrices

5. [**4pts**] Consider the following algorithm:

```
procedure smash(x)
if x > 100 then
   return x/100
else
   return x + smash(10x)
```

Using this algorithm, answer the following questions:

(*i*) Compute `smash(200)`

(*ii*) Compute `smash(50)`

(*iii*) What happens if you try to compute `smash(x)` with $\texttt{x} < 0$?

6. [**4pts**] Use Euclid's algorithm to find the greatest common divisor of 145 and 30. Show all your work.

Chapter 4 – Induction and Recursion

7. **[2pts]** Explain the difference between strong induction and normal induction.

8. **[4pts]** Use induction to prove the following identity holds for all integers $n \geq 1$:

$$1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1$$

Chapter 5 – Counting

9. **[4pts]** Suppose 100 lottery tickets are given out in sequence to the first 100 guests to arrive at a party. Of these 100 tickets, only 12 are winning tickets. The generalized pigeonhole principle guarantees that there must be a streak of at least l losing tickets in a row. Find l.

10. **[2pts]** Use the binomial theorem to expand $(2a - b)^4$.

11. **[2pts]** Find the number of permutations of the word RAXACORICOFALLAPATORIUS.

Chapter 6 – Discrete Probability

12. **[4pts]** In a class of 20 students, 5 are math majors. A group of 4 students are chosen at random.

(i) What is the probability that the group has no math majors?

(ii) What is the probability that the group has at least one math major?

13. **[4pts]** Two shelves on a bookcase contain math books. The top shelf has 6 calculus books and 3 discrete books. The bottom shelf has 5 calculus books and 8 discrete books. A shelf is chosen at random, and from this shelf, a book is chosen at random. It the book happens to be a calculus book, what is the probability that the book was picked from the top shelf.

Chapter 7 – Advance Counting Techniques

14.[**4pts**] Find the solution of the linear homogeneous recurrence relation $a_n = 7a_{n-1} - 6a_{n-2}$ with $a_0 = -1$ and $a_1 = 4$.

15. [**2pts**] Use the principle of inclusion-exclusion to write a formula for $|A \cup B \cup C|$, where A, B, and C are sets.

[**2pts**] How many positive integers not exceeding 100 are divisible by 4, 6, or 10?

Chapter 8 – Relations

16. [**4pts**] Let S denote the set of all students at Lakehead, and define a relation on S as follows:

$$R = \{(a, b) \mid a \text{ and } b \text{ are in a class together}\}.$$

Determine whether the relation is reflexive, symmetric, antisymmetric, and/or transitive. Justify your answer for each property.

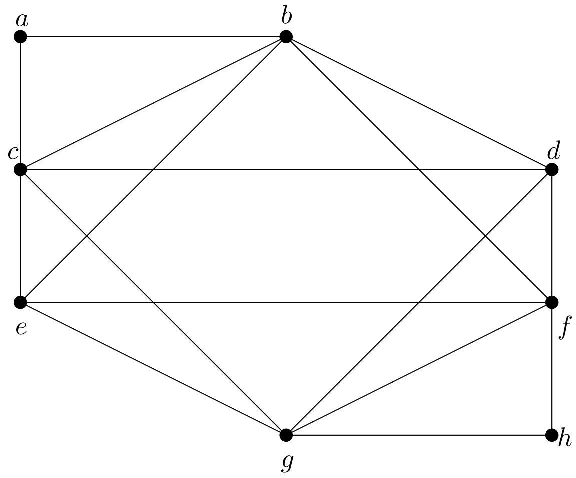
17. [**4pts**] Let $A = \{a, b, c\}$, and let R be a relation on A which is represented by

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

What is the 0-1 matrix of the transitive closure of R?

Chapter 9 – Graphs

18. For the questions below, use the following graph:



(*i*) **[2pts]** Write out the adjacency matrix for the above graph.

(*ii*) **[2pts]** Is the above graph a planar graph? If yes, draw the graph as a planar graph; if no, explain why not.

(*iii*) **[2pts]** Compute the degree of each vertex.

(*iv*) **[2pts]** Does the above graph have a Hamilton path? a Hamilton circuit? If so, write out the path/circuit.

(*v*) **[2pts]** Find the chromatic number of the above graph.

19. **[8pts]** A finite simple graph G is r**-regular** if every vertex has degree r.

  (i) Give an example of a 2-regular graph

  (ii) Which complete bipartite graphs $K_{m,n}$ are r-regular?

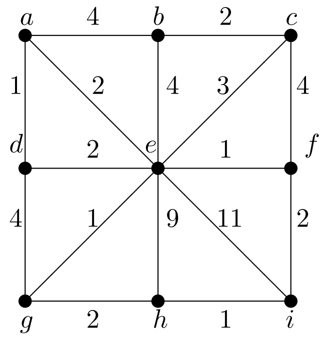
  (iii) Prove that if G is 4-regular with v vertices and e edges, then $2v = e$.

  (iv) Prove that if G is r-regular with r odd, then G must have an even number of vertices.

20. **[2pts]** Draw an example of a graph with an Euler path, but not Euler circuit.

Chapter 10 – Trees

21. Answer the following questions about the weighted graph given below:



(*i*) **[2pts]** Find the shortest path between a and i (you can do this by inspection).

(*ii*) **[3pts]** Use Kruskal's algorithm to find a minimal spanning tree of the above graph. As in class, list the order in which you picked the edges for your spanning tree, and draw your spanning tree.

(*iii*) **[3pts]** (For this question ignore the weights of the graph). Use the breadth-first search to find a spanning tree of the above graph that is rooted at c.

Chapter 11 – Boolean Algebras

22. For the following questions, consider the following Boolean function:

$$F(x, y, z) = xyz + xy\overline{z} + x\overline{y}z.$$

(*i*) **[2pts]** Use a table to express all the values of $F(x, y, z)$.

(*ii*) **[2pts]** Find the dual of $F(x, y, z)$.

(*iii*) **[2pts]** Construct a circuit from inverters, AND gates, and OR gates that produces $F(x, y, z)$ as output.

(*iv*) **[2pts]** Simplify $F(x, y, z)$ using a Karnough map.

23. **[2pts]** Of all the things that you learnt in Math 1281 (Discrete Math), what was your favourite topic/result, and why? (Hint: no wrong answer!)