

NAME: _____

STUDENT NUMBER: _____

MATH 1XX3 (CALCULUS FOR MATH AND STATS II) MIDTERM 1
ADAM VAN TUYL, MCMASTER UNIVERSITY

DAY CLASS

DURATION OF MIDTERM: 50 minutes

MIDTERM

February 11, 2016

THIS MIDTERM PAPER INCLUDES 5 PAGES AND 8 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

SPECIAL INSTRUCTION

- Only the McMaster Standard Calculator (Casio fx 991) is allowed.
- All solutions should be written on the exam.
- The total number of points is 40. **Do all of the 8 questions.**
- For multiple choice questions, full marks will be received for a questions with the correct circled answer. For incorrect answers, part marks are based upon work provided.
- Use the back of pages if you need more room.

Page	Possible	Received
2	10	
3	10	
4	10	
5	10	
Total	40	

1. [5 pts] Which equation does the function $y = e^{-6x}$ satisfy?

a. $y'' - y' - 42y = 0$ b. $y'' - y' + 42y = 0$ c. $y'' + y' - 42y = 0$ d. $y'' + y' + 42y = 0$

2. [5 pts] Solve the differential equation $4y' + 5e^{x+y} = 0$.

a. $y = \frac{5}{4}e^x - C$ b. $y = -\ln\left(\frac{5}{4}e^x - C\right)$ c. $y = \ln\left(\frac{5}{4} + \frac{5}{4}Ce^x\right) - x$ d. $y = \ln\left(\frac{5}{4}x\right) - x$

3. [5 pts] Solve the differential equation $y' = 15 + 5y + 3x + xy$:

a. $y = -3 + Ce^{\frac{x^2}{2} - 5x}$. b. $y = -3 + Ce^{x^2 - 5x}$. c. $y = -3 + Ce^{\frac{x^2}{2} + 5x}$.
d. $y = -3 + Ce^{\frac{x^2}{2} + 5x^2}$ e. $y = -3 + Ce^{x^2 + 5x}$

4. [5 pts] Find the equation of the tangent line to the curve at the point corresponding to the given value of the parameter.

$$x = \cos \theta + \sin 2\theta + 8 \quad y = \sin \theta + \cos 2\theta + 8 \quad \text{when } \theta = \pi.$$

a. $y = \frac{x}{2} + 2$ b. $y = \frac{2}{x}$ c. $y = \frac{x}{2} + \frac{3}{2}$ d. $y = \frac{25}{2} - \frac{x}{2}$ e. None of the above.

5. [5 pts] Alice wants to show her love to Bob by sending him a valentine on her graphing calculator. Which of following curves should Alice use to send Bob a picture of a heart?

a. $r = 4 \cos \theta$. b. $r = 1 - \sin \theta$. c. $r = \cos 3\theta$.

Bob responds by sending Alice a graph to show his love for her is infinite. Which equation does Bob need to use to get an infinity symbol?

a. $r = \theta$ with $\theta \geq 0$. b. $r = \frac{1}{2} + \sin \theta$. c. $r^2 = \cos 2\theta$. [Hint: What is the domain?]

6. [5 pts] Find the area of the region enclosed by one loop of the curve

$$r = 7 \cos 4\theta.$$

[Hint 1: $\cos^2 \theta = \frac{1+\cos 2\theta}{2}$. Hint 2: The curve has 8 loops.]

a. $\frac{49\pi}{11}$ b. $\frac{49\pi}{16}$ c. $\frac{\pi}{4}$ d. $\frac{49}{\pi}$ e. $\frac{\pi}{2}$

7. [5 pts] Math-phobia is a terrible disease that prevents people from doing mathematics! It is known that math-phobia will spread according to the logistic differential equation. In a small town of 3,000 people, 120 people have the disease at the beginning of the week, and 1,600 are afflicted by the end of the week. When will 40% of the population be infected?

8. [5 pts] Consider the two polar curves $r = f(\theta)$ and $r = 2f(\theta)$. How do the lengths of these two curves compare on the interval $\alpha \leq \theta \leq \beta$? Give a reason for your answer.

THE END