Name: ______________________

Student Number: ______________________

Math 1XX3 (Calculus for Math and Stats II) Midterm 2

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Day Class

Duration of Midterm: 50 minutes

Midterm March 17, 2016

This midterm paper includes 6 pages and 9 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instruction

- Only the McMaster Standard Calculator (Casio fx 991) is allowed.
- All solutions should be written on the exam.
- The total number of points is 40. **Do all of the 9 questions**.
- For multiple choice questions, full marks will be received for a questions with the correct circled answer. For incorrect answers, part marks are based upon work provided.
- Use the back of pages if you need more room.

Page	Possible	Received
2	8	
3	8	
4	6	
5	10	
6	8	
Total	40	

1. **[4 pts]** A ball is dropped from a height of 10 m onto a flat surface. Each time the ball hits the surface, it rebounds to 50% of its previous height. Find the total distance the ball travels (assume the ball bounces an infinite number of times).

a. 10 m b. 20 m c. 30 m d. 40 m e. ∞ m

2. **[4 pts]** Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^4}.$$

[Hint: What's an upper bound for $\arctan x$?]

a. conditionally convergent b. absolutely convergent c. divergent

3. **[4pts]** Test for convergence or divergence (use any test): $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+4}$.

4. **[4pts]** Test for convergence or divergence (use any test): $\sum_{n=2}^{\infty} \frac{5}{n(\ln n)^2}$.

5. **[4pts]** Test for convergence or divergence (use any test): $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$.

[Hint: You may want to use the fact that $\lim_{n \to \infty} n^{\frac{1}{n}} = 1$.]

6. **[2 pts]** What is wrong with following argument that $0 = \pi$:

$$\begin{aligned} 0 &= 0 + 0 + 0 + 0 + \cdots \\ &= (\pi - \pi) + (\pi - \pi) + (\pi - \pi) + \cdots \\ &= \pi + (-\pi + \pi) + (-\pi + \pi) + (-\pi + \pi) + \cdots \\ &= \pi + 0 + 0 + \cdots = \pi \end{aligned}$$

7. **[5 pts]** Let $C > 0$ be any positive real number. Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(Cx)^n}{n!}.$$

a. $R = C, I = (-C, C)$ b. $R = 0, I = [0]$ c. $R = C, I = [-C, C]$ d. $R = \infty, I = (-\infty, \infty)$.

8. **[5 pts]** The Maclaurin Series of $\sin x$ is given by

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

Use this fact to find the values of r and s for which

$$\lim_{x \to 0} \left(\frac{\sin 3x}{x^3} + \frac{r}{x^2} + s \right) = 0.$$

9. [**8 pts**] The following statements are all **FALSE**. For each statement, give an example to show that the statement is false.

(a) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

(b) If $\sum_{n=1}^{\infty} a_n^2$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

(c) A series that converges must converge absolutely.

(d) If $0 \leq a_n \leq b_n$ for all $n \geq 0$ and if $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ also diverges.

THE END