MATH 1B03: Midterm 1 - VERSION 1
Instructor: Adam Van Tuyl
Date: October 3, 2018 7:00PM
Duration: 75 min.

Name: SOLUTIONS ID #: ______________

Instructions:

This test paper contains **20** multiple choice questions printed on both sides of the page. The questions are on pages 2 through 10. Scrap paper is available for rough work. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCIES TO THE ATTENTION OF THE INVIGILATOR.

Select the one correct answer to each question and ENTER THAT ANSWER INTO THE SCAN CARD PROVIDED USING AN HB PENCIL. You are required to submit this booklet along with your answer sheet. HOWEVER, NO MARKS WILL BE GIVEN FOR THE WORK IN THIS BOOKLET. Only the answers on the scan card count for credit. Each question is worth 1 mark. The test is graded out of 20. There is no penalty for incorrect answers.

NO CALCULATORS ALLOWED.

Computer Card Instructions:

IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED. YOUR TEST RESULTS DEPEND UPON PROPER ATTENTION TO THESE INSTRUCTIONS.

The scanner that will read the answer sheets senses areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid.

- Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet **MUST** be signed in the space marked SIGNATURE.
- Mark your student number in the space provided on the sheet on Side 1 **and fill the corresponding bubbles underneath**. Your student number **MUST** be 9 digits long. If you have a student number that is 7 digits, begin your student number with 00 (two zeroes).
- Mark only **ONE** choice (A, B, C, D, E) for each question.
- Begin answering questions using the first set of bubbles, marked "1".

1. Which equation is **NOT** linear in x_1, x_2 and x_3.

(a) $x_1 + x_2 + x_3 = 2018.$

(b) $\boxed{\sqrt{2018x_1} + \pi^{2018}x_2 + e^{2018x_3} = 42.}$

(c) $(\sin(2018))x_1 + (\ln(2018))x_3 = 0.$

(d) $2018x_1 + (e^{\sin 2018})x_2 + (\log_{10} 2018)x_3 = 2018.$

(e) $-x_1 - 2x_2 - 3x_3 - 4 = 0.$

$\sqrt{2018x_1}$ and e^{2018x_3} are not linear terms

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2. Which of the following matrices are in row echelon form?

(i) $\begin{bmatrix} 1 & 3 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 & -2 \\ 0 & 0 & 1 & -9 & 8 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 1 & -5 & 0 & 0 \\ 0 & 0 & 0 & \pi & 6 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

(iv) $\begin{bmatrix} 2018 & 0 \\ 0 & 42 \end{bmatrix}$ (v) $\begin{bmatrix} 0 & 1 & 0 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 1 & -8 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 11 & 0 \end{bmatrix}$

(a) i) only

(b) i) and iv) only

(c) iv) only

(d) All of them

(e) None of them

i) is in row echelon form
ii) is not in row echelon form since in the second row, the first nonzero entry is not one.
iii) is not in row echelon form since there is an entry below the leading one of the first row
iv) is not in row echelon form since the leading entries are not 1 (it is in echelon form)
v) not in row echelon form since leading entry of 3rd row is 6

3. What is the reduced row echelon form of the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix}.$$

(a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 8/3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix} \sim \begin{bmatrix} 12 & 24 & 36 & 48 \\ 12 & 12 & 18 & 24 \\ 12 & 12 & 12 & 16 \\ 12 & 12 & 12 & 12 \end{bmatrix} \sim \begin{bmatrix} 12 & 24 & 36 & 48 \\ 0 & 12 & 18 & 24 \\ 0 & 0 & 6 & 8 \\ 0 & 0 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 12 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the following reduced row echelon form. Solve the system.

$$\begin{bmatrix} 1 & -9 & 0 & 0 & 5 & -2 \\ 0 & 0 & 1 & 0 & -5 & 4 \\ 0 & 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) $\begin{aligned} x_1 &= 9 - 5s - 2t \\ x_2 &= s \\ x_3 &= 5s + 4t \\ x_4 &= 3s + 6t \\ x_5 &= t \end{aligned}$ (b) $\begin{aligned} x_1 &= -2 + 9s - 5t \\ x_2 &= s \\ x_3 &= 4 + 5t \\ x_4 &= 6 - 3t \\ x_5 &= t \end{aligned}$ (c) $\begin{aligned} x_1 &= 2 \\ x_2 &= 0 \\ x_3 &= -4 \\ x_4 &= -6 \\ x_5 &= 0 \end{aligned}$ (d) no solution

We are given

$$\begin{aligned} x_1 - 9x_2 + 5x_5 &= -2 \\ x_3 - 5x_5 &= 4 \\ x_4 + 3x_5 &= 6 \end{aligned}$$

So x_5 and x_2 are free. This means the answer is either (a) or (b). Note that

$$x_4 = 6 - 3x_5 = 6 - 3t$$

So, this means the sol^n must be (b)

(You can substitute each $x_1, \ldots, x_5$ into the eqn's and check they are solns)

5. The **rank** of a matrix A is the number of leading 1's in the reduced row echelon form of A. What is the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & a & 1-a & a^2+1 \\ 1 & 2-a & -1 & -2a^2 \end{bmatrix}$$

$$A \sim \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & a & 1-a & a^2+1 \\ 0 & -a & 0 & -2a^2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & a & 1-a & a^2+1 \\ 0 & 0 & 1-a & a^2-2a^2+1 \end{bmatrix}$$

(a) 0

(b) 1

(c) 2

(d) 3

If $a=1$, number of leading 1's is 2

If $a=0$, number of leading 1's is 3. So answer is (e)

$\boxed{\text{(e)}}$ Not enough information; answer will depend upon the value of a.

6. Suppose that A is a 3×2017 matrix, B is a 2017×4 matrix, C is a 3×4 matrix, $C^T AD$ is a 4×2018 matrix, and $D^T BC^T$ is a 2018×3 matrix. What is the size of the matrix D?

(a) 4×3

(b) 3×2018

(c) 2017×4

(d) 2018×2017

$\boxed{\text{(e) } 2017 \times 2018}$

Note that B is 2017×4 and C^T is 4×3.

So D is $m \times n$ matrix, and D^T is an $n \times m$ matrix.

Thus $\underset{n \times m}{D^T}\ \underset{2017 \times 4}{B}\ \underset{4 \times 3}{C^T}$ is a 2018×3 matrix if this matrix multiplication is defined. This happens if $m = 2017$ and $n = 2018$.

So D is a 2017×2018 matrix

7. Suppose that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$ and when multiplied together, they commute (i.e., $AB = BA$). Which of the following must necessarily be true?

(a) $a = 0$

$\boxed{\text{(b) } A \text{ is a diagonal matrix}}$

(c) c can be any number

(d) $d = 0$

(e) none of the above

We have $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2b \\ 0 & 2d \end{bmatrix}$

and $\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2c & 2d \end{bmatrix}$.

Thus $\begin{bmatrix} 0 & 2b \\ 0 & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2c & 2d \end{bmatrix}$. Hence $2b = 0$ and $2c = 0$.

This forces $c = b = 0$, and thus $A = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$,

i.e. A is diagonal

8. If A, B, C and D are invertible matrices of the same size and

$$(3DA^{-1}B)^{-1} = C,$$

which of the following must be A?

(a) $\frac{1}{3}BCD$

* $\boxed{\text{(b) } 3BCD}$

* (c) $\frac{1}{3}D^{-1}BC$

(d) $3BD^{-1}C$

(e) $\frac{1}{3}BCD^{-1}$

$(3DA^{-1}B)^{-1} = C$ expand out left side

$3^{-1}B^{-1}(A^{-1})^{-1}D^{-1} = C$ simplify left side

$\frac{1}{3}B^{-1}AD^{-1} = C$

$3B(\frac{1}{3}B^{-1}AD^{-1})D = 3BCD$ multiply on left by $3B$ and right ~~by~~ by D to solve for A

So $A = 3BCD$

9. Compute A if $(B+3A)^{-1} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$.

(a) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$

(e) $\boxed{\begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}}$

Note $(B+3A) = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1} = \frac{1}{8-9}\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$

So $3A = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} - B = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -6 & 3 \\ 0 & -3 \end{bmatrix}$

Thus $A = \frac{1}{3}\begin{bmatrix} -6 & 3 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$

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10. Which one of the following statements is not equivalent to the others?

(a) A is invertible.

(b) $A\mathbf{x} = \mathbf{0}$ has only one solution.

(c) The reduced row echelon form of A has a row of zeros.

(d) A is a product of elementary matrices.

(e) $A\mathbf{x} = \mathbf{b}$ is consistent for every $n \times 1$ matrix $\mathbf{b}$.

If A is invertible, then ~~A is~~ the reduced row echelon form of A is I_n which has no row of zeros

11. Let k_1, k_2, k_3 be a nonzero numbers. What is the inverse of the matrix

$$\begin{bmatrix} 0 & 0 & k_1 \\ 0 & k_2 & 0 \\ k_3 & 0 & 0 \end{bmatrix}?$$

Quickest way to see this is to note that

$$\begin{bmatrix} 0 & 0 & k_1 \\ 0 & k_2 & 0 \\ k_3 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1/k_3 \\ 0 & 1/k_2 & 0 \\ 1/k_1 & 0 & 0 \end{bmatrix} = I_3$$

(a) $\begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & \frac{1}{k_1} \\ 0 & \frac{1}{k_2} & 0 \\ \frac{1}{k_3} & 0 & 0 \end{bmatrix}$ (c) $\boxed{\begin{bmatrix} 0 & 0 & \frac{1}{k_3} \\ 0 & \frac{1}{k_2} & 0 \\ \frac{1}{k_1} & 0 & 0 \end{bmatrix}}$

(d) $\begin{bmatrix} 0 & 0 & k_3 \\ 0 & k_2 & 0 \\ k_1 & 0 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 0 & -\frac{1}{k_1} \\ 0 & k_3 & 0 \\ -k_2 & 0 & 0 \end{bmatrix}$

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12. $A = \begin{bmatrix} 2 & 0 & 1 \\ 6 & 4 & 2 \end{bmatrix}$ is transformed into $B = \begin{bmatrix} 6 & 4 & 2 \\ 14 & 8 & 5 \end{bmatrix}$ by two elementary row operations, the first operation of which involves swapping rows. What are the corresponding elementary matrices E_1 and E_2?

$\boxed{\text{(a) } E_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } E_2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}}$

(b) $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and $E_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $E_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ and $E_2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

(d) $E_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e) $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

We are given that

$B = E_2 E_1 A$.

a swap

not a swap.

Looking at choices of E_1, only (a) contains an elementary matrix for E_1 that is a row swap. So answer is A

13. There are real numbers a, b, and c such that graph of the polynomial

$$p(x) = ax^2 + bx + 3$$

passes through the three points $(1, c), (-1, 6)$ and $(2, 3)$. What is abc (i.e., find a, b, and c, and then compute the product abc)?

(a) $\boxed{-4}$
(b) -2
(c) 2
(d) 4
(e) None of the above.

We are given

$$\left.\begin{aligned} a\cdot 1 + b\cdot 1 + 3 &= c \\ a - b + 3 &= 6 \\ 4a + 2b + 3 &= 3 \end{aligned}\right\} \Rightarrow \left[\begin{array}{ccc:c} 1 & 1 & -1 & -3 \\ 1 & -1 & 0 & 3 \\ 4 & 2 & 0 & 0 \end{array}\right]$$

Corresponding SLE

Solving this system gives $a = 1$, $b = -2$, $c = 2$. So $abc = \boxed{-4}$

14. What is the standard matrix for the linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$f\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 + 5x_2 - x_3 \\ 4x_1 - x_2 + x_3 \\ 3x_1 + 2x_2 - x_3 \end{bmatrix}$$

(a) $\begin{bmatrix} 3 & 4 & 3 \\ 5 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 5 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ $\boxed{\text{(e)} \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}}$

To find the matrix, we need to compute $f\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$, $f\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right)$ and $f\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right)$

$$f\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix} \quad f\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix} \quad f\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

So standard matrix is $A = \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$

15. The following three commands are inputted into Matlab. What is the final output?

```
A = [1 2; 3 4; 5 6]
B = A'
prod(B(:,3))
```

Input 1: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

Input 2: $B = A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

Input 3 $\to$ take product of entries in third column, so $5 \times 6 = 30$

$\boxed{\text{(a) } 30}$ (b) 48 (c) $\begin{matrix} 5 \\ 6 \end{matrix}$ (d) $\begin{matrix} 2 \\ 4 \\ 6 \end{matrix}$ (e) 2018

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16. Which of the following statements are true?

(1) Multiplying a row of an augmented matrix through by 2018 is an acceptable elementary row operation. TRUE

(2) If a linear system has three equations in two unknowns, then the system is always inconsistent. FALSE

$\to$ $\left.\begin{aligned} x_1 + x_2 &= 1 \\ 2x_1 + 2x_2 &= 2 \\ 3x_1 + 3x_2 &= 3 \end{aligned}\right\}$ three equations, two unknowns, that is consistent.

(a) (1) is false and (2) is false.

$\boxed{\text{(b) (1) is true and (2) is false.}}$

(c) (1) is false and (2) is true.

(d) (1) is true and (2) is true.

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17. Which of the following statements are true?

FALSE

(1) If a matrix is in row echelon form, then it is also in reduced row echelon form.

(2) Suppose that a homogeneous system of 2017 linear equations has 2018 unknowns. Then it has an infinite number of solutions. TRUE

(a) (1) is false and (2) is false.

(b) (1) is true and (2) is false.

$\boxed{\text{(c) (1) is false and (2) is true.}}$

(d) (1) is true and (2) is true.

18. Which of the following statements are true?

    (1) If A and B are $n \times n$ matrices, then $(A+B)^T = A^T + B^T$. TRUE

    (2) If A and B are $n \times n$ matrices, then $(AB)^T = A^T B^T$. FALSE.

    Correct formula $(AB)^T = B^T A^T$

    (a) (1) is false and (2) is false.

    $\boxed{\text{(b) (1) is true and (2) is false.}}$

    (c) (1) is false and (2) is true.

    (d) (1) is true and (2) is true.

19. Which of the following statements are true?

    (1) If A is a $n \times n$ invertible matrix, and E is an elementary $n \times n$ matrix, then EA^T is invertible. TRUE

    (2) If A is an $n \times n$ matrix and $A\mathbf{x} = \mathbf{b}$ has exactly one solution for all $n \times 1$ matrices $\mathbf{b}$, then A is invertible. TRUE

    (a) (1) is false and (2) is false.

    (b) (1) is true and (2) is false.

    (c) (1) is false and (2) is true.

    $\boxed{\text{(d) (1) is true and (2) is true.}}$

$A+B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

So $A+B$ is upper triangular, but neither A or B have this prop

20. Which of the following statements are true?

    (1) If A and B are $n \times n$ matrix such that $A+B$ is upper triangular, then A and B are upper triangular. FALSE

    (2) If $f : \mathbb{R}^{2018} \longrightarrow \mathbb{R}^2$ is a matrix transformation, then there is a 2018×2 matrix A such that $f(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^{2018}$. FALSE

    The matrix A should be $\underline{2 \times 2018}$.

    $\boxed{\text{(a) (1) is false and (2) is false.}}$

    (b) (1) is true and (2) is false.

    (c) (1) is false and (2) is true.

    (d) (1) is true and (2) is true.

END OF TEST PAPER