MATH 1B03: Midterm 2 - VERSION 1
Instructor: Adam Van Tuyl
Date: November 7, 7:30PM
Duration: 75 min.

Name: ______________________________________ **ID #:** ________________

Instructions:

This test paper contains **20** multiple choice questions printed on both sides of the page. The questions are on pages 2 through 10. Scrap paper is available for rough work. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCIES TO THE ATTENTION OF THE INVIGILATOR.

Select the one correct answer to each question and ENTER THAT ANSWER INTO THE SCAN CARD PROVIDED USING AN HB PENCIL. You are required to submit this booklet along with your answer sheet. HOWEVER, NO MARKS WILL BE GIVEN FOR THE WORK IN THIS BOOKLET. Only the answers on the scan card count for credit. Each question is worth 1 mark. The test is graded out of 20. There is no penalty for incorrect answers.

NO CALCULATORS ALLOWED.

Computer Card Instructions:

IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED. YOUR TEST RESULTS DEPEND UPON PROPER ATTENTION TO THESE INSTRUCTIONS.

The scanner that will read the answer sheets senses areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid.

- Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet **MUST** be signed in the space marked SIGNATURE.
- Mark your student number in the space provided on the sheet on Side 1 **and fill the corresponding bubbles underneath**. Your student number **MUST** be 9 digits long. If you have a student number that is 7 digits, begin your student number with 00 (two zeroes).
- Mark only **ONE** choice (A, B, C, D, E) for each question.
- Begin answering questions using the first set of bubbles, marked “1”.

1. Find the determinant of the following matrix:

$$A = \begin{bmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 0 & 3 \\ 0 & 5 & 0 & -6 \end{bmatrix}.$$

   (a) -20
   (b) -5
   (c) 5
   (d) 20
   (e) 2018

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2. Suppose that A is an $n \times n$ matrix such that $A^2 = I_n$. Which statement is true about A?

   (a) $\lambda = 0$ is an eigenvalue of A.
   (b) $\det(A^2) = 2\det(A)$.
   (c) $\det(A) = \pm 1$.
   (d) A is not invertible.
   (e) Not enough information provided.

3. Consider the matrix

$$A = \begin{bmatrix} 0 & -2 & -1 \\ 5 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}.$$

What is $(\text{adj}(A))_{2,3}$, i.e., what is the value in position $(2,3)$ of the adjoint matrix $\text{adj}(A)$?

(a) -5

(b) 0

(c) 1

(d) 2

(e) 10

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4. Given the matrices

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} a & b & a \\ c & d & c \\ 2 & -1 & 3 \end{bmatrix}$$

and the fact that $\det(A) = 2018$, what is $\det(B)$?

(a) -2018

(b) 0

(c) 4036

(d) 2018

(e) Not enough information provided.

5. Suppose that A is an invertible matrix with $\det(A) = 6$, B is an invertible matrix with $\det(B) = 3$, and C is an invertible matrix with $\det(C) = 2$. Furthermore, each matrix has size 2×2. Find the determinant of the matrix:

$$5A^{-1}B^{T}\mathrm{adj}(C)$$

(a) 75
(b) $\frac{5}{2}$
(c) $\frac{5}{6}$
(d) $\frac{25}{2}$
(e) 25

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6. Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}.$$

(a) -3 and -7
(b) 3 and -7
(c) -3 and 7
(d) 3 and 7
(e) 3 (with multiplicity two)

7. The matrix
$$A = \begin{bmatrix} -1 & -1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$
is similar to the diagonal matrix
$$D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$
What is the matrix P that diagonalizes A?

(a) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & -1 & 1 \\ 0 & -\frac{1}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(e) None of the above

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8. Which of the following vectors are eigenvectors of the matrix
$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(a) $\begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 2018 \\ 2018 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ (e) All of them

9. The following commands are inputted into Matlab:

```
a=@(i,j) i*j;
A = zeros(2,4);
for i=1:2
    for j=1:4
        A(i,j)=a(i,j);
    end
end
A
```

What is the final output?

(a)
```
1    2    3    4
2    4    6    8
```
(b)
```
1    2
2    4
3    6
4    8
```
(c) `2018`

(d)
```
1    2    3    4
2    3    4    5
```
(e)
```
1    -2    3    -4
2    3    -4    5
```

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10. The matrix

$$\begin{bmatrix} a & 4 \\ -3 & 7 \end{bmatrix}$$

has eigenvalues $\lambda = 1$ and $\lambda = 5$. What is a?

(a) $a = -7$.
(b) $a = 1$.
(c) $a = -1$.
(d) $a = 7$.
(e) Not enough information provided.

11. What is

$$\left(\frac{1}{i}\right)^{2018}$$

(a) 0 (b) $-i$ (c) -1 (d) i (e) 1

12. You are given two complex numbers written in polar coordinate form:

$$\begin{aligned} z_1 &= \sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right) \\ z_2 &= \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)). \end{aligned}$$

What is z_1/z_2? (Note: the following numbers are not in polar coordinate form.)

(a) $1+i$
(b) $-1+i$
(c) $\sqrt{2}+i$
(d) $\sqrt{2}-i$
(e) $2+i$

13. The matrix A is a 3×3 matrix such that A is invertible. Which of the following <u>cannot</u> be the characteristic polynomial of A?

(a) $\lambda^3+3\lambda^2+3\lambda+4$
(b) $\lambda^3+6\lambda^2+3\lambda+3$
(c) $\lambda^3+10\lambda^2+3\lambda+2$
(d) $\lambda^3+15\lambda^2+3\lambda+1$
(e) $\lambda^3+21\lambda^2+3\lambda$

14. Brontosauruses and tyrannosaurus rexes fight for resources on the continent of Pangaea. Suppose that the population of each species is modeled by the system of differential equations

$$\begin{aligned} y_1' &= 4y_1 - 5y_2 \\ y_2' &= -2y_1 + y_2 \end{aligned}$$

where y_1 is the size of the brontosaurus population and y_2 is the size of the tyrannosaurus rex population at time x. Which equation models the brontosaurus's population? [Below, the c_1 and c_2 are constants that depend upon the initial population sizes.]

(a) $y_1 = -5c_1e^{-x} + c_2e^{6x}$

(b) $y_1 = c_1e^{6x}$

(c) $y_1 = -5c_1e^{6x} + c_2e^{-x}$

(d) $y_1 = 2c_1e^{6x} + c_2e^{-x}$

(e) $y_1 = 2018e^{6x} + 2018e^{-x}$

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15. This question continues the previous question. Suppose that at time $x = 0$, we have $y_1(0) = 6,000$ and $y_2(0) = 20,000$. What is the tyrannosaurus rex population at time $x = 1$?

(a) 14000

(b) 2018

(c) $2000e^6$

(d) $-10000e^6 + 16000e^{-1}$

(e) $4000e^6 + 16000e^{-1}$

16. Which of the following statements are true?

    (1) If A is an $n \times n$ matrix, then the determinant of A is the product of the entries on the main diagonal of the matrix.

    (2) If B is the matrix that results when two rows of A are interchanged, then $\det(B) = -\det(A)$.

    (a) (1) is false and (2) is false.

    (b) (1) is true and (2) is false.

    (c) (1) is false and (2) is true.

    (d) (1) is true and (2) is true.

17. Which of the following statements are true?

    (1) If A is an invertible matrix, then $\det(A)A^{-1} = \text{adj}(A)$.

    (2) If A is an $n \times n$ matrix, then a nonzero vector $\mathbf{x}$ in $\mathbb{R}^n$ is called an eigenvector if there is a scalar λ such that $A\mathbf{x} = \lambda\mathbf{x}$.

    (a) (1) is false and (2) is false.

    (b) (1) is true and (2) is false.

    (c) (1) is false and (2) is true.

    (d) (1) is true and (2) is true.

18. Which of the following statements are true?

    (1) If λ is an eigenvalue of the matrix A, then the matrix $(\lambda I - A)$ is invertible.

    (2) If λ is an eigenvalue of the matrix A, then λ is also an eigenvalue of the reduced row echelon form of A.

    (a) (1) is false and (2) is false.

    (b) (1) is true and (2) is false.

    (c) (1) is false and (2) is true.

    (d) (1) is true and (2) is true.

19. Which of the following statements are true?

    (1) For every eigenvalue of a matrix A, the geometric multiplicity of the eigenvalue is greater than or equal to the algebraic multiplicity of the eigenvalue.

    (2) If A is an $n \times n$ matrix with n distinct eigenvalues, then A is diagonalizable.

    (a) (1) is false and (2) is false.

    (b) (1) is true and (2) is false.

    (c) (1) is false and (2) is true.

    (d) (1) is true and (2) is true.

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20. Which of the following statements are true?

    (1) If $\mathbf{x}' = A\mathbf{x}$ and $\mathbf{y}' = A\mathbf{y}$, then $\mathbf{x} = \mathbf{y}$.

    (2) There is a real 3×3 matrix with no real eigenvalues.

    (a) (1) is false and (2) is false.

    (b) (1) is true and (2) is false.

    (c) (1) is false and (2) is true.

    (d) (1) is true and (2) is true.

END OF TEST PAPER