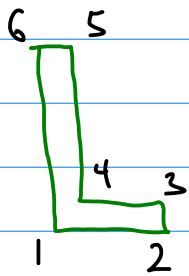Applications to Computer Graphics

Today's lecture: linear algebra and computer graphics

Example   Consider the letter "L" as a graphic

← letter "L" determined by
6 pts or vertices, and information
about which vertices are connected

Store positions in a matrix:

$$D = \begin{array}{c} \text{Vertex} \\ \text{x-coord} \\ \text{y-coord} \end{array} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \left[\begin{array}{cccccc} 0 & 1 & 1 & 0.25 & 0.25 & 0 \\ 0 & 0 & 0.25 & 0.25 & 2 & 2 \end{array}\right] \end{array}$$

Octave commands to draw "L"

```
A = [0,1,1,.25,.25,0,0];
B=[0,0,.25,.25,2,2,0];
plot(A,B);axis([-2 3 -2 3]);
```

last column goes back
to first spot

plots a line from $\begin{bmatrix} x_i \\ y_i \end{bmatrix}$ to $\begin{bmatrix} x_{i+1} \\ y_{i+1} \end{bmatrix}$

Problem How do you make an italic L?

An italic L is formed by performing a shear operation

A shear operation uses the matrix

$$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$ where the k controls the size of the shear

Ex Suppose k = 0.25. Describe L after the shear.

We need new coordinates

$$\begin{bmatrix} 1 & 0.25 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 & 0.25 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0.25 & 2 & 2 & 0 \end{bmatrix}$$
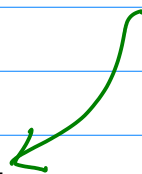
$$= \begin{bmatrix} 0 & 1 & 1.0625 & 0.3125 & 0.75 & .5 & 0 \\ 0 & 0 & .25 & .25 & 2 & 2 & 0 \end{bmatrix}$$

Octave code:

```
S=[1 .25; 0 1];
D=[A;B];
T=S*D;
plot(T(1,:),T(2,:));axis([-2 3 -2 3]);
```

T(1,:) ← first row
T(2,:) ← second row

The italic L looks a little too wide.

Rescale it by "shrinking" all the x-coordinates

$$\begin{bmatrix} 0.8 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1.0625 & 0.3125 & 0.75 & .5 & 0 \\ 0 & 0 & .25 & .25 & 2 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & .8 & .85 & .25 & .6 & .4 & 0 \\ 0 & 0 & .25 & .25 & 2 & 2 & 0 \end{bmatrix}$$

Octave Code

```
R = [.8 0;0 1];
Q = R*S*D;
plot(Q(1,:),Q(2,:));axis([-2 3 -2 3]);
```
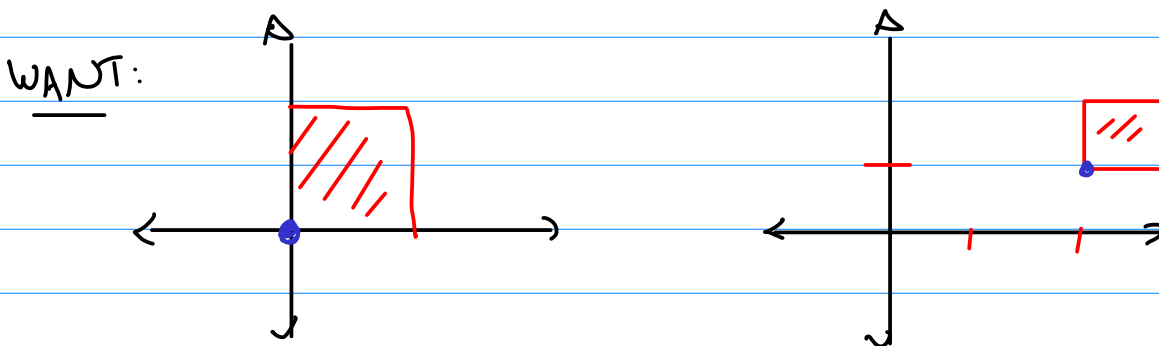
Remark: with matrix operations, can perform
    shears, flips, and rotations
    (see Section 1.9)

Current tools do not allow translations!

Homogeneous Coordinates

Translations: move an object

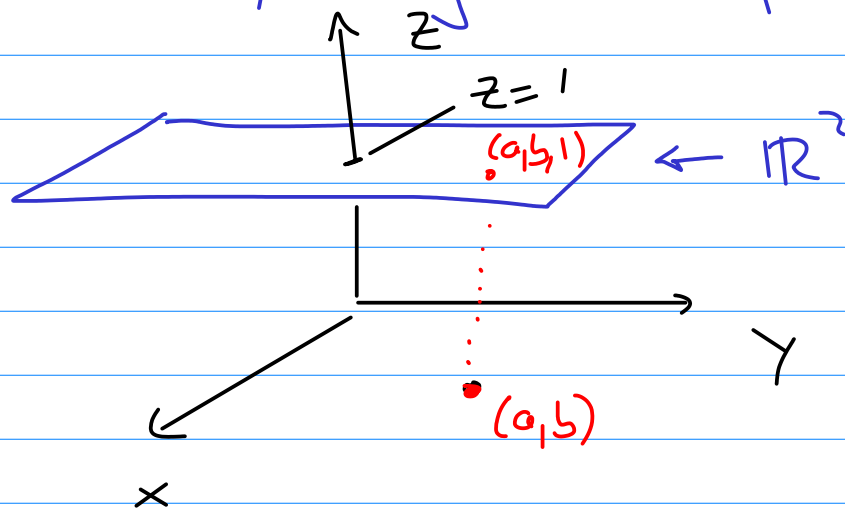Ex Translate unit square two units right and one up

WANT:

However, no matrix A will work!

We want $A\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

BUT $A\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ for any A!

We can "cheat" by viewing $\mathbb{R}^2$ as a plane in $\mathbb{R}^3$

z

$z = 1$

$(a,b,1)$ $\leftarrow \mathbb{R}^2$

Y

(a,b)

x

Each point (a,b) can be identified with homogeneous coordinat $(a,b,1)$

Consequence:

A translation $\begin{bmatrix} x \\ y \end{bmatrix} \longmapsto \begin{bmatrix} x+h \\ y+k \end{bmatrix}$

can be written in homogeneous coordinates as

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \longmapsto \begin{bmatrix} x+h \\ y+k \\ 1 \end{bmatrix}$$

Matrix form

$$\begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+h \\ y+k \\ 1 \end{bmatrix}$$

<u>Problem</u> Move our italic L two units right and
one unit down

Octave code:

added a row of ones

```
HQ=[Q;[1 1 1 1 1 1 1]];
G = [1 0 2; 0 1 -1; 0 0 1];
K=G*HQ;
plot(K(1,:),K(2,:));axis([-2 3 -2 3]);
```

← translation matrix

<u>Thm</u> Let $T:\mathbb{R}^2 \rightarrow \mathbb{R}^2$ be any linear transformation,
and let A be the standard matrix of T. Then
the partitioned matrix

$$\begin{bmatrix} A & 0 \\ \hline 0 & 1 \end{bmatrix}$$

is the linear transformation
with respect to homogeneous
coordinates

we move the points in $\mathbb{R}^2$
into the "copy" of $\mathbb{R}^2$
in $\mathbb{R}^3$ with all $z=1$

Key ideas: * use Octave to plot graphics
* visualize linear transformations
* homogeneous coordinates