



MATH 1B03 C01 and C02: Midterm 1 (Version 2 - Early Writes/SAS)

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Date: October 2, 2019 5:30PM

Duration: 75 min.

First name (please write as legibly as possible within the boxes)

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Instructions:

(YOUR RESULTS DEPEND UPON PROPER ATTENTION TO THESE INSTRUCTIONS.)

- Fill in your name and student ID **neatly** in the box above.
- This test paper contains **16** multiple choice questions and **2** short answer questions printed on both sides of the page. The questions are on pages 2 through 10. Page 11 is a blank page for calculation, and Page 12 is the bubble sheet for the multiple choice questions. Scrap paper is also available for rough work.
- For Questions 1 through 16, select the one correct answer to each question and enter that answer on the bubble sheet.
- For Questions 17 and 18, write your answer directly in this test booklet.
- The midterm is graded out of 21. Questions 1 through 16 are worth 1 point each, Question 17 is worth 2 points, and Question 18 is worth 3 points.
- **NO CALCULATORS ALLOWED.**
- **YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE TEST IS COMPLETE. LET THE INVIGILATOR KNOW OF ANY DISCREPANCIES.**



1. Which equation is **NOT** a linear equation in x_1, x_2 and x_3 .

a) $x_1 + x_2 + x_3 = 2019$.

b) $\sin(2019x_1) + (\cos(2019))x_3 = 0$.

c) $\sqrt{2019}x_1 + \pi^{2019}x_2 + e^{2019}x_3 = 42$.

d) $2^{2019}x_1 + 6x_2 + (\log_{10} 11)x_3 = 2019$.

e) $-x_1 - 2x_2 - 3x_3 - 4 = 0$.

2. Which of the following matrices are in *row echelon form*?

row echelon

i)
$$\begin{bmatrix} 1 & 3 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 & -2 \\ 0 & 0 & 1 & -9 & 8 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

not echelon

ii)
$$\begin{bmatrix} 0 & 1 & -5 & 0 & 0 \\ 0 & 0 & 0 & \pi & 6 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

echelon only

iii)
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

echelon only

iv)
$$\begin{bmatrix} 42 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

v)
$$\begin{bmatrix} 0 & 1 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

reduce to row echelon,
so row echelon

a) i), ii) and v) only

b) All of them

c) None of them

d) i) and v) only

e) v) only



3. How many solutions does the following system of linear equations have?

$$-2x_1 + 3x_2 - 3x_3 = -9$$

$$3x_1 - 4x_2 + x_3 = 5$$

$$-5x_1 + 7x_2 - 4x_3 = -14$$

- a) None
b) One
c) Two
d) 2019

e) Infinitely many

$$\begin{bmatrix} -2 & 3 & -3 & -9 \\ 3 & -4 & 1 & 5 \\ -5 & 7 & -4 & -14 \end{bmatrix} \sim \begin{bmatrix} -2 & -3 & -3 & -9 \\ 3 & -4 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

← from this shape, we see we will have an infinite # of solⁿs

4. Suppose that the augmented matrix of a system of linear equations has been placed into the following reduced row echelon form:

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 5 & -3 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

If q, r are arbitrary elements of \mathbb{R} , then the set of solutions for this systems is described by

a) $x_1 = -3$
 $x_2 = 0$
 $x_3 = 1$
 $x_4 = 0$
 $x_5 = 2$

(b) $x_1 = -3 - 2q - 5r$
 $x_2 = q$
 $x_3 = 1 + r$
 $x_4 = r$
 $x_5 = 2$

(c) $x_1 = 3 + 2q + 5r$
 $x_2 = q$
 $x_3 = 1 - r$
 $x_4 = r$
 $x_5 = 2$

(d) $x_1 = 2q + 5r$
 $x_2 = q$
 $x_3 = -1 + r$
 $x_4 = r$
 $x_5 = -2$

Same as V.1



5. The rank of a matrix A is the number of leading 1's in the reduced row echelon form of A . What is the rank of the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1-a & 1 & a^2+1 \\ 1 & -a & 3 & a^2+1 \end{bmatrix}$$

a) 0

b) 1

c) 2

d) 3

e) Not enough information; answer will depend upon the value of a .

$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1-a & 1 & a^2+1 \\ 1 & -a & 3 & a^2+1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1-a & 1 & a^2+1 \\ 0 & 1-a & 1 & a^2+1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1-a & 1 & a^2+1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Regardless of value of a , always have two leading 1's

6. You are given the following three matrices

$$A = \begin{bmatrix} 5 & -6 & 4 & -4 \\ -8 & 9 & -2 & 3 \\ -4 & 7 & 3 & -1 \end{bmatrix}, B = \begin{bmatrix} -8 & 9 & -2 & 3 \\ 8 & -5 & 4 & -1 \\ -2 & 5 & -3 & 2 \end{bmatrix}, C = \begin{bmatrix} -4 & 9 \\ 6 & -5 \\ -9 & 3 \\ -1 & 5 \end{bmatrix}$$

What matrix multiplication will yield a 2×2 matrix?

a) $CABC$

b) $C^T A^T BC$

c) $A^T BCC^T$

d) $BC^T A^T B$

e) $ABA^T C$



7. If A , B , C and D are invertible matrices of the same size and

$$C^{-1}BC^T A^2 A^T = D$$

which of the following must be B ?

a) $(A^{-1})^T(A^2)^{-1}C(C^T)^{-1}D$

b) $CD(A^{-1})^T(A^2)^{-1}(C^T)^{-1}$ ← Same as V.1.

c) $CDA^T A^2 C^T$

d) $CDA(C^T)^{-1}$

e) None of the above

8. Compute A if $(B + A)^{-1} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$.

a) $\begin{bmatrix} 6 & 3 \\ 3 & 6 \end{bmatrix}$

b) $\begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 3 & 6 \\ 0 & -6 \end{bmatrix}$

e) $\begin{bmatrix} -6 & 3 \\ 0 & -3 \end{bmatrix}$

$$B+A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1} = \frac{1}{8-9} \begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$$

$$\text{So } A = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} - B = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 3 \\ 0 & -3 \end{bmatrix}$$



9. What are the entries of the first row of A^{-1} if

$$A = \begin{bmatrix} 0 & 2 & 2 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

a) $-\frac{1}{2}, 1, 0$

b) $0, 1, \frac{1}{2}$

c) $1, 0, -1$

d) $\frac{1}{2}, 0, 1$

e) $-\frac{1}{2}, 0, 1$

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & 1 & 0 \\ 1 & 0 & -1 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix}$$

← details worked out in V.1

10. Which one of the following statements is not equivalent to the others?

a) A is invertible.

b) $Ax = 0$ has no non-trivial solution.

c) The reduced row echelon form of A is I_n .

d) A is not a product of elementary matrices.

e) $Ax = b$ is consistent for every $n \times 1$ matrix b .

← all other statements equivalent



11. The system of 5 equations in 4 unknowns $Ax = B$ has solutions

$$x = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} -3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

If performing row operations on the augmented matrix $[A|B]$ can produce the following matrix

$$\left[\begin{array}{cccc|c} 2 & 2 & 3 & -1 & 1 \\ 0 & 1 & 5 & 0 & a \\ 12 & 0 & 7 & -6 & b \\ 8 & 3 & 4 & -4 & c \\ 22 & 0 & 11 & -11 & d \end{array} \right]$$

Since $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ is a solⁿ,

what is $a + d$?

- a) -123
- b) -8
- c) -10
- d) 10
- e) 123

$$\begin{bmatrix} 0 & 1 & 5 & 0 \\ 22 & 0 & 11 & -11 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \Rightarrow \begin{array}{l} 1 = a \\ -11 = d \end{array}$$

So $a + d = 1 - 11 = -10$

12. In Matlab, suppose that we have defined a $n \times n$ matrix M , and we want make a new matrix where each (i, j) -th entry is the square of the (i, j) -th entry of the matrix M . Which command could accomplish this?

- a) square(M)
- b) $M(1,1)^2, M(1,2)^2, \dots, M(n,n)^2$
- c) M^2
- d) $M.^2$
- e) $M * M'$



The following questions are all **TRUE-FALSE** questions.

13. Which of the following statements are true?

- (1) Multiplying a row of an augmented matrix through by 0 is not an acceptable elementary row operation. **TRUE**
- (2) If a linear system has two equations in three unknowns, then the system is always inconsistent. **FALSE**

a) (1) is false and (2) is false.

b) (1) is true and (2) is false.

c) (1) is false and (2) is true.

d) (1) is true and (2) is true.

14. Which of the following statements are true?

(1) If A and B are $n \times n$ matrices, then $\text{tr}(AB) = \text{tr}(A)\text{tr}(B)$. **FALSE**

(2) If A and B are $n \times n$ matrices, then $\text{tr}(A+B) = \text{tr}(A) + \text{tr}(B)$. **TRUE**

a) (1) is false and (2) is false.

b) (1) is true and (2) is false.

c) (1) is false and (2) is true.

d) (1) is true and (2) is true.

Same as V.1



15. Which of the following statements are true?

- (1) If A is any invertible $n \times n$ matrix and E is an $n \times n$ elementary matrix, then EA is invertible. **TRUE**
- (2) If E_1 and E_2 are two $n \times n$ elementary matrices, then $E_1 + E_2$ is also an elementary matrix. **FALSE**
- a) (1) is false and (2) is false.
- b) (1) is true and (2) is false.
- c) (1) is false and (2) is true.
- d) (1) is true and (2) is true.

16. Which of the following statements are true?

- (1) If A and B are $n \times n$ matrix such that A and B are symmetric, then $A+B$ is symmetric. **TRUE**
- (2) If A is an $n \times m$ matrix, then AA^T is an $n \times n$ symmetric matrix. **TRUE**
- a) (1) is false and (2) is false.
- b) (1) is true and (2) is false.
- c) (1) is false and (2) is true.
- d) (1) is true and (2) is true.

$$S = S^T \quad A = (S^T A)^T \quad A = S^T A$$

$$S^T S = I \quad S = S^{-1} \quad A = S^T A$$



The following two questions are **SHORT ANSWER QUESTIONS**. Write your answer for each question directly into the space provided. You will be graded for both your answer and your explanation.

17. A square $n \times n$ matrix A is called **idempotent** if $A^2 = A$. Show that if A is an idempotent matrix, then the matrix $(I_n - A)$ is also an idempotent matrix.

Same as V.1

18. Suppose that A is an $n \times n$ matrix such that $Ax = \mathbf{0}$ has a non-trivial solution. Prove that $A^{2019}x = \mathbf{0}$ has must also have a non-trivial solution.

Let \vec{w} be the nontrivial solⁿ to $A\vec{x} = \vec{0}$, i.e. $A\vec{w} = \vec{0}$ but $\vec{w} \neq \vec{0}$. Then \vec{w} is also a solⁿ to $A^{2019}\vec{x} = \vec{0}$ since

$$A^{2019}\vec{w} = A^{2018}(A\vec{w}) = A^{2018}\vec{0} = \vec{0}.$$

So $A^{2019}\vec{x} = \vec{0}$ has a nontrivial solⁿ



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