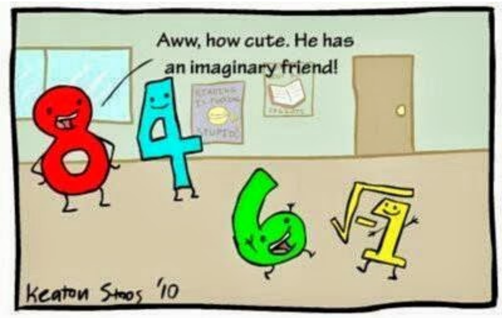
$\boxed{\text{Lecture 1}}$ 1.A Intro to $\mathbb{R}$ and $\mathbb{C}$



$\mathbb{R}$ = all real numbers
$\mathbb{C}$ = all complex numbers

Recall $x^2 = -1$ has no real roots
Let i be the sol^n to this equation, so $i^2 = -1$

Complex numbers ← all numbers of the form $a+bi$

$$\mathbb{C} = \{a+bi \mid a, b \in \mathbb{R}\}$$

eg $2+3i$, $3-\pi i$, 2π, $-17i$

$\boxed{\text{Arithmetic of } \mathbb{C}}$

addition: $(a+bi)+(c+di) = (a+c)+(b+d)i$

subtraction: $(a+bi)-(c+di) = (a-c)+(b-d)i$

multiplication: $(a+bi)(c+di) = ac + adi + bci + bdi^2$
$= ac + (ad+bc)i - bd$
$= (ac-bd) + (ad+bc)i$

division: $\dfrac{a+bi}{c+di} = \dfrac{(a+bi)}{(c+di)} \cdot \dfrac{(c-di)}{(c-di)} = \dfrac{ac+bd}{c^2+d^2} + \dfrac{(bc-ad)i}{c^2+d^2}$

Notation Write $\mathbb{F}$ if a property holds for both $\mathbb{R}$ and $\mathbb{C}$.

May call $\mathbb{F}$ a field

(Properties of F) Let $\alpha, \beta, \gamma \in F$

- $\alpha+\beta = \beta+\alpha$ (addition commutes)
- $(\alpha+\beta)+\gamma = \alpha+(\beta+\gamma)$ (associative)
- $\alpha+0=\alpha$
- for all $\alpha \in F$, there is $\beta \in F$ such that $\alpha+\beta=0$

- $\alpha\beta = \beta\alpha$ (mult. commutes)
- $\alpha(\beta\gamma)=(\alpha\beta)\gamma$
- $\alpha \cdot 1 = \alpha$
- For all $\alpha \in F$, $\alpha \neq 0$, there is $\beta \in F$ such that $\alpha \cdot \beta = 1$

Notation
- $-\alpha$ denotes the β such that $\alpha+(-\alpha)=0$
- $1/\alpha$ denotes the β such that $\alpha \cdot 1/\alpha = 1$

Def^{n} If $\alpha \in F$, $n \in \mathbb{N}$, $\alpha^n = \underbrace{\alpha \cdot \alpha \cdots \alpha}_{n}$

Def^{n} Elements of F are called scalars

Lists/n-tuples

A <u>list</u> of length n is an ordered collection n elements written as

$$(x_1, x_2, x_3, \ldots, x_n)$$ ← also called an n-tuple

<u>Ex</u> (lists $\neq$ sets) In a set, order does not matter. In a list, order matters

$\{1,2,3\} = \{3,1,2\}$ ← sets $\qquad (1,2,3) \neq (3,1,2)$ ← as lists

<u>Def$^{\underline{n}}$</u> $F^n = \{ (x_1, x_2, \ldots, x_n) \mid x_i \in F \}$ ← all n-tuples of length n.

<u>Ex</u> $\mathbb{R}^2 = \{ (x,y) \mid x, y \in \mathbb{R} \}$ ← pts on the plane

<u>Remark</u> In 1B03, used column notation, i.e. $\mathbb{R}^2 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x, y \in \mathbb{R} \right\}$

In 2R03 we use the list notation:

$\underline{\text{Def}^{n}}$: If $(x_1, \dots, x_n) \in F^n$, call x_j is the j^{th}-coordinate

$\boxed{\text{Operations on } F^n}$

__Addition__ If $(x_1, \dots, x_n), (y_1, \dots, y_n) \in F^n$, then

$$(x_1, \dots, x_n) + (y_1, \dots, y_n) = (x_1 + y_1, x_2 + y_2, \dots, x_n + y_n)$$

__Scalar multiplication__: If $c \in F$ and $(x_1, \dots, x_n) \in F^n$, then

$$c(x_1, \dots, x_n) = (cx_1, \dots, cx_n)$$

__Notation__. · The book/course may write $x \in F^n$ ~~&~~ pay attention to context. This tells us
$x = (x_1, \dots, x_n)$

· $0 \in F^n$ means $0 = (0, 0, \dots, 0)$ as a list

__Ex__ Show $\lambda(x+y) = \lambda x + \lambda y$ for all $x, y \in F^n$, $\lambda \in F$

__Proof__ Since $x, y \in F^n$, $x = (x_1, \dots, x_n)$ and $y = (y_1, \dots, y_n)$

So $\lambda(x+y) = \lambda((x_1,\dots,x_n)+(y_1,\dots,y_n))$ add x & y

$= \lambda((x_1+y_1, x_2+y_2, \dots, x_n+y_n))$ scalar mult

$= (\lambda(x_1+y_1), \lambda(x_2+y_2), \dots, \lambda(x_n+y_n))$

$= (\lambda x_1+\lambda y_1, \lambda x_2+\lambda y_2, \dots, \lambda x_n+\lambda y_n) \leftarrow$ prop of F

$= (\lambda x_1, \lambda x_2, \dots, \lambda x_n) + (\lambda y_1, \dots, \lambda y_n)$

$= \lambda(x_1, \dots, x_n) + \lambda(y_1, \dots, y_n)$

$= \lambda x + \lambda y$ $\blacksquare$

<u>Remark</u>

- F stands for <u>field</u>
- $\mathbb{R}$ and $\mathbb{C}$ are fields, but so is $\mathbb{Q}$
- For this course, only focus on $\mathbb{R}$ and $\mathbb{C}$
- (Learn about fields in 3GR3)

Key ideas

* review of complex numbers
* intro to F and F^n