NAME: Solutions

STUDENT NUMBER: ____________

MATH 3V03 - Midterm 1

McMaster University

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Instructions: Answer all questions in the space provided. If you need more room, answer on the back of the page. Where appropriate, you must provide clear explanations.

You are *not* allowed to use a calculator. If a question involves a calculation you may leave it in an unexpanded form, e.g., you can write 5^4 instead of 625.

If doubt exists as to the interpretation of any question, you are urged to submit with the answer paper a clear statement of any assumptions made.

Page	Possible	Received
2	10	
3	10	
4	10	
5	10	
Total	40	

1. **[10 pts]** In the space provided, match each term with its definition. (Note: There are more definitions than terms.)

Terms.

G Pseudograph

L Multigraph

A Tree

N Forest

H Bipartite graph

F k-regular

B Girth

D Diameter

E Euler circuit

J Hamilton cycle

Definitions.

A. A connected graph that contains no subgraph isomorphic to a cycle.

B. The length of the shortest cycle in the graph.

C. An edge of G whose removal disconnects G.

D. The maximum distance between any two vertices of G.

E. A circuit that contains every edge of G.

F. A graph where every vertex has degree k.

G. A graph where multiple edges and loops are allowed.

H. A graph with chromatic number ≤ 2.

I. A graph which has chromatic number k.

J. A cycle in G that contains every vertex of G.

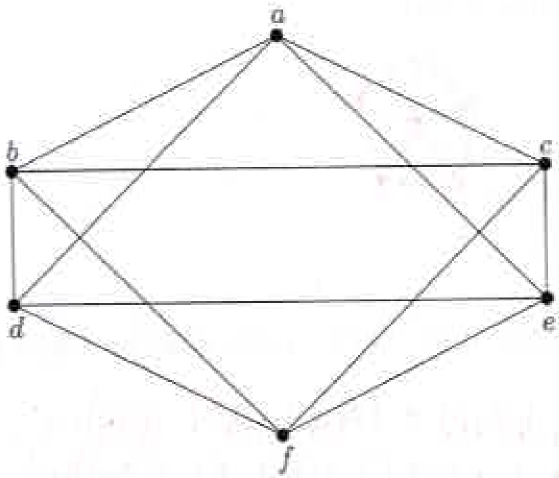
K. A trail whose endpoints are the same vertex.

L. A graph where multiple edges are allowed but not loops.

M. A trail ~~that contains every edge and every vertex of G~~ with the property that no vertex is repeated.

N. A graph that contains no cycle.

2. Use the following graph to answer the questions found below.



(i) **[2pts]** Find the degree of each vertex.

(ii) **[2pts]** Compute $\chi(G)$.

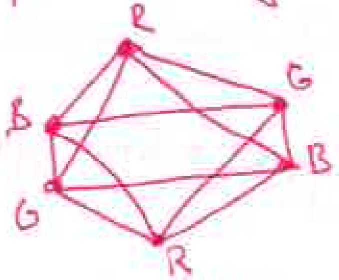
(iii) **[2pts]** Find a spanning tree ~~of the~~ above graph.

* (iv) **[2pts]** Does the above graph have an Euler circuit? If so, write out the path/circuit, or explain why no such path/circuit exists. Repeat the question for a Hamilton ~~circuit~~. cycle

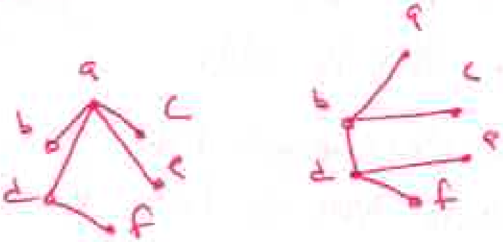
* (v) **[2pts]** ~~Find a decomposition of G into cycles.~~ Decompose G into three isomorphic cycles.

(i) every vertex has degree 4

(ii) $\chi(G)=3$



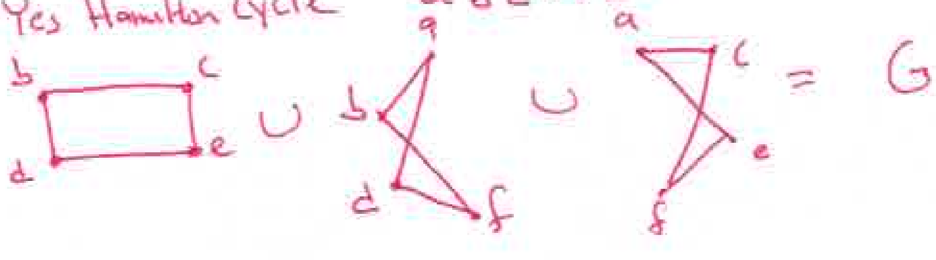
(iii) many possibilities



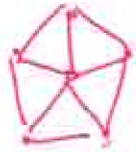
(iv) Yes Euler Circuit abcedfcadbfea

Yes Hamilton Cycle abdfeca

(v)



3. **[2pts]** Draw the following two graphs: (i) $K_{2,3}$, (ii) W_5.

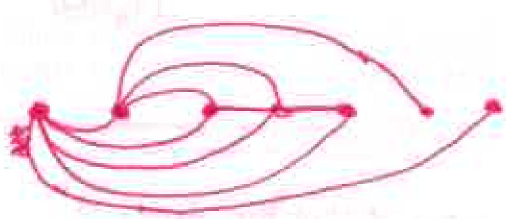
(i)  (ii) 

4. **[4pts]** Is the sequence $(5,4,4,3,2,1,1)$ graphic? If yes, draw a graph with this degree sequence. If no, justify your answer.

$(5,4,4,3,2,1,1)$ graphic $\Leftrightarrow$ $(3,3,2,1,0,1) = (3,3,2,1,1,0)$ graphic
$\Leftrightarrow$ $(2,1,0,1,0) = (2,1,1,0,0)$ graphic

Because $(2,1,1,0,0)$ is graphic, e.g 
original graph is graphic

Here is a graph with this degree seq.



5. **[4pts]** Prove that if $m \geq 3$ and m is odd, then the complete bipartite graph $K_{3,m}$ cannot be decomposed into two isomorphic subgraphs.

By the Handshaking Theorem, $K_{3,m}$ has $\frac{3 \cdot m + m \cdot 3}{2} = 3m$ edges.
If m is odd, then $3m$ is odd.
Thus, $K_{3,m}$ cannot be decomposed into two isomorphic subgraphs,
because each graph would have to have the same # of edges,
i.e. $3m$ would have to be divisible by 2.

6. **[2pts]** Give an example of a graph that is critical with $\chi(G) = k$, but G contains no subgraph isomorphic to K_k.

C_5:  $\chi(C_5)=3$ but it has no subgraph isomorphic to K_3

7. **[4pts]** Suppose that G is a graph with $p = 10$ vertices and $q = 16$ edges. Suppose that each vertex of G either has degree 3 or degree 4. How many vertices have degree 3 and how many have degree 4?

Let p_3 = # of vertices of degree 3
p_4 = # " " " " 4.

So $p_3+p_4=p=10$ and by the Handshaking Theorem, $3p_3+4p_4=2\cdot 16=32$.

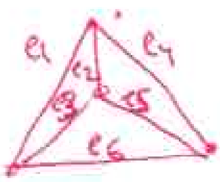
So $p_3=10-p_4$. Thus $3(10-p_4)+4p_4=32$
$\Rightarrow 30+p_4=32$
$\Rightarrow p_4=2$

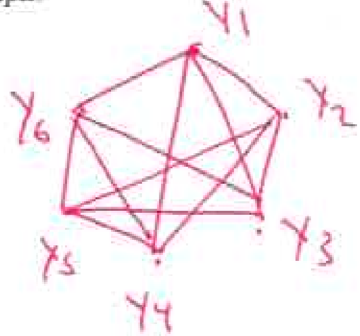
Thus $\boxed{p_4=2 \text{ and } p_3=8}$

8. **[4pts]** Given a graph $G = (V_G, E_G)$ on p vertices and q edges $E_G = \{e_1, \ldots, e_q\}$, we can make a new graph $L(G) = (V_{L(G)}, E_{L(G)})$, called the **line graph**, as follows. Let $V_{L(G)} = \{y_1, \ldots, y_q\}$ and let

$$E_{L(G)} = \{y_iy_j \mid \text{edge } e_i \text{ is adjacent to } e_j\}.$$

Show that $L(K_4)$, the line graph of K_4, is a regular graph.

K_4 

$L(K_4)$ 

From $L(K_4)$, we see every vertex is degree 4.
So $L(K_4)$ is 4-regular.