

Corrections and clarifications to Ian Stewart's *Galois Theory*, Third Edition

Many of these mistakes were discovered by Ken Ribet's Math 114 class a year ago; a few more were sent to me by Adam van Tuyl of Lakehead University, Ontario. I am adding still more as I and my class find them.

P.3, third line from bottom: "Theorem 5" should be "Theorem 24.5".

P.7, next-to-last display: Cross out the $\frac{1}{2}$ after the $=$ sign.

P.8: The label "(1.2)" should be on the line with the first displayed equation, $y^3 + py + q = 0$, not lower down. And in the second displayed equation, " $p = \dots$ ", the numerator should simply be $3b - a^2$. (In effect, the $-2a^3$ shown should be $-2a^2$, which simplifies with the preceding term.)

P.9, line after first display: " $u + v = q$ " should be " $u + v = -q$ ".

P.10, second display: In both summands, the -18 should be 18 .

P.11: In first display, $3a$ should be $3a^2$. In second display, $a^2/4$ should be $a^3/8$. In the middle of the page, line immediately before "We choose", $+2uy+$ should be $+2uy^2+$. In line immediately after "We choose", $\frac{q}{2}\sqrt{2u}$ should be $q/(2\sqrt{2u})$, and the same correction applies in each of the last two displays in the page. Finally, in display (1.11), $2p$ should be $2p^2$.

P.13, second display (the 4th-power expression): This should be $(\alpha_1 + \alpha_2 - \alpha_3 - \alpha_4)^2$.

P.14, Exercise 1.3: Change "its prime factorization is of the form" to "it can be written in the form"; and after "where" insert "the p_i are primes and".

P.14, Exercise 1.4: After "Prove" add "without assuming Cardano's Formula". Also, as in the above correction to p.10, both -18 's should be 18 .

P.15, Exercise 1.8: After "when" add " p and q are real numbers such that".

P.15, Exercise 1.11: The first sentence should be "Let $P(n)$ be the number of length- n strings of 0's and 1's such that all 1's that occur (if any) occur in groups of three or more."

P.20, line 5: Delete the last two equations (those defining $-\infty \times n$ and $(-\infty)^2$) and put in the equation $(-\infty) + (-\infty) = -\infty$.

P.20, proof of Proposition 2.3: In the display, change $a_{n-1}t^{n-1}$ to $a_n t^n$. In the rest of the proof, change all occurrences of n to m .

P.20, first line of last paragraph: "Proposition 2.2" should be Proposition 2.3.

P.21, 8th line from bottom: "that using" should be "then using".

P.25, sentence after last display, beginning "Because ...": The justification given there is incorrect. By doing the preceding calculation carefully one can, however, verify that the function is continuous as a function of two variables. But since this concerns analysis, which is not the subject of this course, don't worry about it; you may simply assume the continuity condition holds.

P.26, display slightly below middle of page, " $q(t) =$ ": The exponent n in the first summand should be $n-1$, and the last two coefficients, a_1 and a_0 , should be a_2 and a_1 respectively.

P.32, top two lines: You might find what he means clearer if you change "This" to "This result, which we will now state", and change "in which f is assumed to be linear" to "which is essentially what we get if we put $t - \alpha$ for f in this result".

P.32, 1st line of 2nd paragraph of proof of Proposition 3.1: Change "for all polynomials of" to "whenever g has".

P.34, 3rd, 4th lines of proof of Lemma 3.5: There are four occurrences of K ; the first three should be k .

P.35, two lines above Theorem 3.9: Change "this chapter" to "this section".

P.36, Definition 3.10: After "A polynomial" add "of positive degree". (This change brings Stewart's definition closer to the usual one, though they are still not equivalent. It is needed for consistency with what Stewart will do on p.40.)

- P.37, line 3: Delete this line (beginning “5.”), since this example is not valid after the above correction.
- P.37, Theorem 3.12: In the statement, change “*Any nonzero polynomial*” to “*Any polynomial of positive degree*”; in the second line of the proof change “= 0 or 1” to “= 1”, and in the third line of proof change “ h and K ” to “ h and k ”.
- P.38, Theorem 3.16: In the first line of the theorem, after “*factorization of polynomials*” add “*of positive degree*”. In the proof, delete the second and third sentences, “If all ... Otherwise ...”. (These concern the possibility that some of the irreducible polynomials may be constant, which is not possible under the corrected definition of irreducibility.) Note that as a result, the argument shows that $r = s$. (Stewart uses this fact in the proof of Corollary 3.18 on p.40.)
- P.42, last sentence of §3.4: “Lemma 13” should be Lemma 19.13.
- P.42, first line of §3.5: “is that” should be “in”.
- P.42, last line: $1 \leq a \leq n - 1$ should be $1 \leq a \leq n$. (This only makes a difference for $n = 1$, but having the correct value for $\varphi(1)$ is essential to results such as that of Exercise 3.13.)
- P.43, line after second display: “Exercise 3.1” should be Exercise 3.9.
- P.43, beginning of 6th line before Example 3.23: “is irreducible” should be “is reducible”.
- P.43, 4th line before Example 3.23: $\mathbb{Q}[t]$ should be $\mathbb{Z}[t]$.
- P.45, third line after Definition 3.26: “Figure 1.1” should be Figure 3.1.
- P.46, Theorem 3.28, first line: Change “*a polynomial*” to “*a nonzero polynomial*”.
- P.49, first display: The final “+5” should be “-5”.
- P.51, last line: After “Moreover” add “it is easy to check that L is a subring of \mathbb{C} , hence”.
- P.53: In the line above the last display, $\mathbb{C}(X)$ should be $\mathbb{Q}(X)$. In the last display, the denominator should have variables x_1, \dots, x_n , like the numerator. On the next line, “and y_j ” should be dropped. And on the final line, “For a proof see” should be “You will prove this as”.
- P.56, Exercise 4.9: In the first line, $\mathbb{C}(X)$ should again be $\mathbb{Q}(X)$. In the last line, $\mathbb{Q}[t]$ should be $\mathbb{Q}[t_1, \dots, t_n]$.
- P.59, top line: By “constant multiple” Stewart means “scalar multiple”, i.e., the result of multiplying by a member of K .
- P.59, beginning of line 9: Change “contrary to the definition” to “and of lower degree than p (since the highest-degree terms of p and q were the same, and so cancelled), contradicting our assumption that p had smallest degree among such polynomials”.
- P.60, statement of Lemma 5.8: The two congruences “ $a_1 + a_2 \equiv b_1 + b_2$ ” and “ $a_1 a_2 \equiv b_1 b_2$ ” should be $a_1 + b_1 \equiv a_2 + b_2$ and $a_1 b_1 \equiv a_2 b_2$. Likewise, in the first display in the proof, a_2 should in both places be b_1 , and vice versa.
- P.62, line 5: “Chapter 16” should be Chapter 17.
- P.62, beginning of last line of Theorem 5.11: “ $K(t) \rightarrow K$ ” should be “ $K(t) \rightarrow K(\alpha)$ ”.
- P.62, 2nd line of Theorem 5.12: “*isomorphic to $K[t]/\langle m \rangle$* ” should be “*isomorphic to $K[t]/\langle m \rangle : K$* ”.
- P.63, Lemma 5.14: Move this to p.68, end of §6.1.
- P.70, two lines above middle display: delete the words “unless $a = b = 0$ ”. And in that display, the left-hand side, which is shown as ending “ $]:\mathbb{Q}$ ”, should end “ $:\mathbb{Q}]$ ”.
- P.72, line after Definition 6.10: “Exercise 6.12” should be Exercise 6.11.
- P.73, Exercise 6.8: “if ∂p and $[L:K]$ are coprime” should be “if ∂p does not divide $[L:K]$ ”.
- P.75, 8th lines from bottom, et seq.: The author uses, here and throughout the chapter, the wording (which I suppose is British school-usage) “draw a circle, centre O ...” where we would say “draw a circle with center O ...”.
- P.77, Figure 7.2 is poorly drawn: p_1 and p_2 are supposed to be the centers of the two circles.
- P.82, top line: “contracting” should be “constructing”.

- P.84, Exercise 7.15: After “a regular hexagon” add “of side $AB = 1$ ”.
- P.84, first line of Exercise 7.16: In the angles named, remove the denominators “3”.
- P.90, end of third line: “of maps” should be “of bijective maps”.
- P.90, end of first line of 2nd paragraph: “(1) and (2)” refer to the statements at the top of the preceding page.
- P.92, first line of item 3: “Example 8” should be Example 6.8.
- P.93, near end of top line: The “is” (after $\subseteq L$) should be deleted.
- P.94, line after last display: “Exercise 7.1” should be Exercise 8.1.
- P.95, 4 lines above next-to-last display: After “generated” add “over \mathbb{C} ”.
- P.95, line after next-to-last display: Change “the lexicographic ordering” to “an appropriate ordering”, and at the end of the line change “Exercise 8.2” to “Exercise 8.4”.
- P.96, last line of Definition 8.8: Change $K_{j+1} = K_j(\alpha_j)$ to $K_j = K_{j-1}(\alpha_j)$.
- P.96, end of next-to-last line: Change $\alpha_j^{p_1 \cdots p_l}$ to $\alpha_j^{p_{l+1} \cdots p_n}$ ($l = 0, \dots, k$). And on the last line, change the formula shown to “ $\beta_0 \in K_j$ and $\beta_l^{p_l} \in K_j(\beta_{l-1})$ ”.
- P.97, second line after first display: $N \subseteq \mathbb{A}_n$ should be $N \supseteq \mathbb{A}_n$.
- P.98, middle: The end-of-proof sign \square should be one line higher.
- P.98, “PROOF OF THEOREM 10” should be “PROOF OF THEOREM 8.10”.
- P.98, line after last display: $\sigma(\alpha_1) \notin K$ should be $\sigma(\alpha_1) \neq \alpha_1$.
- P.99, two lines above 3rd display: Add “Thus $K_1 = K(\alpha) \subseteq K(\delta)$. But $K(\alpha)$ and $K(\delta)$ are both of degree 2 over K , so in fact they are equal.”
- P.99, line above the “ \square ” in middle of page: “Lemma 11” should be Lemma 8.11.
- P.100: Before reading Theorem 8.12, read Definition 15.1 on p.153 (ignoring the phrase “in \mathbb{C} ”). Back in Theorem 8.12, the condition that $F(t)$ “can be solved by radicals” means that each of its roots t_i is contained in a radical extension of K , in the sense of that definition.
- P.100, third line above Definition 8.15: “Exercise 15.14” should be Exercise 15.11.
- P.100, line after Definition 8.15: Change “every radical extension has finite height” to “the height of every radical extension is defined”. (By definition it will, indeed, be finite, but that isn’t the point.)
- P.101, first full line: $kp + ql$ should be $qp + lk$.
- P.101, third line after first display: After “linear factors” add “over some larger field”.
- P.101, second display and preceding sentence: In the sentence, “ P and P_j ” should be “ P_j and P_k ”, and in the display, $cP(t)$ should be $cP_k(t)$.
- P.101, 4th line before last display: ∂p should be ∂P . Beginning of line after that display: “When $\partial p =$ ” should be “Since $\partial P =$ ”
- P.101, last 3 lines, and p.102, first two lines: Don’t take this too seriously, since he has been assuming since the middle of the preceding page that we are given a radical extension R' of K containing L , with no questions asked about where we got it.
- P.102, statement of Lemma 8.17: Here R and M are assumed as on the preceding page, in the sentence before the last three lines.
- P.102, two lines before third display: In the first of these, after “Therefore,” add “up to multiplication by nonzero elements of M ”. You should expect it to take some nontrivial thinking to verify the assertion of this sentence. In the next line, α^s should be $x_s \alpha^s$.
- P.102, last display: under the Π , the \mathbb{S} should be \mathbb{S}_n .

P.102, before last line insert:

Lemma 8.18½

Suppose $L \subseteq M$ are fields, and p a prime such that L contains a primitive p th root of unity ζ . Suppose moreover that $\alpha \neq 0$ and x_0, \dots, x_{p-1} are elements of M such that L contains the p elements

$$\begin{aligned} & x_0 + \alpha x_1 + \alpha^2 x_2 + \dots + \alpha^{p-1} x_{p-1} \\ & x_0 + (\zeta\alpha) x_1 + (\zeta\alpha)^2 x_2 + \dots + (\zeta\alpha)^{p-1} x_{p-1} \\ & \vdots \\ & x_0 + (\zeta^{p-1}\alpha) x_1 + (\zeta^{p-1}\alpha)^2 x_2 + \dots + (\zeta^{p-1}\alpha)^{p-1} x_{p-1}. \end{aligned}$$

Then each of the elements $x_0, \alpha x_1, \alpha^2 x_2, \dots, \alpha^{p-1} x_{p-1}$ also lies in L . Hence, if $x_1 = 1$, then α and each of the elements x_0, \dots, x_{p-1} lies in L .

PROOF. For any $m = 0, \dots, p-1$, consider the sum in L of the first element in the above list, ζ^{-m} times the second element, ζ^{-2m} times the third, etc. through $\zeta^{-(p-1)m}$ times the last. Recalling that $1 + \zeta + \dots + \zeta^{p-1} = 0$, we can verify that each ‘‘column’’ in the resulting sum adds up to zero except the column beginning $\alpha^m x_m$, which adds up to $(p-1)\alpha^m x_m$. So that element lies in L , hence so does $\alpha^m x_m$, as claimed.

If $x_1 = 1$, then the $m = 1$ case of this observation gives $\alpha \in L$, hence the fact that each element $\alpha^m x_m$ lies in L implies that each element x_m does. \square

P.103, line before last paragraph of proof: Change ‘‘Therefore’’ to:

Now if we take the equation $m(x) = 0$ and write x as above in terms of powers of α and coefficients in K , and consider the result as an equation satisfied by α , this will have the form $f(\alpha) = 0$ where $f(t) \in K[t]$. Hence $f(t)$ is divisible by the minimal polynomial of α , which is $t^p - a$. Hence all the roots of that polynomial, namely $\alpha, \zeta\alpha, \dots, \zeta^{p-1}\alpha$ where ζ is a primitive p th root of unity, are also roots of $f(t)$. Now by the way we just defined $f(t)$, this means all the elements $x_0 + \zeta^j\alpha + x_2(\zeta^j\alpha)^2 + \dots + x_{p-1}(\zeta^j\alpha)^{p-1}$ ($j = 0, \dots, p-1$) are roots of $m(t)$. Hence these lie in L , and so Lemma 8.18½ shows that

P.103, first line of last paragraph of proof: ‘‘Also, α ’’ should be ‘‘Also, α^p ’’.

P.103, two lines above ‘‘ \square ’’: ‘‘Exercise 8.6’’ should be Exercise 8.11.

P.104, first line of Exercise 8.4: ‘‘Exercise 2.12’’ should be Exercise 8.3.

P.105, Exercise 8.8: $(\alpha_1 - \alpha_2)^2$ should just be $\alpha_1 - \alpha_2$.

P.105, Exercise 8.10: Change the second line to ‘‘show that $\alpha_{\sigma(1)} + \alpha_{\sigma(2)} - \alpha_{\sigma(3)} - \alpha_{\sigma(4)}$ is a Ruffini radical for all $\sigma \in \mathbb{S}_4$.’’

P.105, Exercise 8.12: In parts (c) and (d), after ‘‘the Galois group of’’ add ‘‘any finite algebraic extension’’.

P.109, second line of Lemma 9.5: \supseteq should be \supseteq .

P.109, line before last display: ‘‘Theorem 16’’ should be Theorem 5.16.

P.109, last display: The first α_1 should be σ_1 . (The display should read $j_1: K(\sigma_1) \rightarrow K'(\alpha_1)$.)

P.111, middle: Drop the ‘‘ \square ’’ and put it on the next page, line 5 (i.e., end of the first paragraph).

P.111, 5th line from bottom: Cross out the sentence beginning ‘‘Furthermore’’.

P.112, line 2: ‘‘(8.1)’’ should be (9.1).

P.113, end of statement of Lemma 9.13: ‘‘ $\Sigma[t]$ ’’ should be $K[t]$.

P.133, last line before Theorem 12.1: Replace the words ‘‘that is’’ by ‘‘and as we saw in (8.2) (p.93),’’.

P.137, third-from-last display: Note that the British write ‘‘2.4’’ where we write ‘‘2·4’’ and vice versa. This will crop up in a few more places later on.

P.138, Figure 13.1, lower right-hand corner: This should be labeled $-i\xi$ (the negative of the label on the upper left-hand corner).

- P.139, upper block-display: In the third and fourth lines, the comma before the final τ should be deleted; i.e., σ^2, τ and σ^3, τ should be $\sigma^2\tau$ and $\sigma^3\tau$ respectively.
- P.145, beginning of second display: “ $1 + H_0$ ” should be $1 = H_0$.
- P.147, first display: Write in the margin *Here and on the next page he composes permutations from left to right*.
- P.147, second display: On this line and the preceding, “(1b)” should be “(2b)” (three occurrences in all).
- P.153, last display: “ $(j \geq 2)$ ” should be “ $(j \geq 1)$ ”.
- P.154, 3rd and 4th lines after Definition 15.2: Change “it is pointless to expect everything expressible by the same radicals to be inside the splitting field” to “those radicals need not themselves lie in the splitting field”.
- P.155, change last sentence of proof of Lemma 15.4 to, “Since α_i is a member of a radical sequence for a subfield of M , so is β_{ij} . By taking a union of such sequences, we get a radical sequence for M .”
- P.155, line after Lemma 15.5: “We” should be “When $K = \mathbb{Q}$ we”.
- P.156: Delete the line beginning “Again,” after the end of the proof at top of page.
- P.161, Exercise 15.1(b): “ $)$ ”)” should just be “ $)$ ”.
- P.165, Example 16.4, part 5: $\mathbb{Q}[t]$ should be $\mathbb{Q}(t)$. (In contrast to part 4, where $\mathbb{Q}[t]$ is correct.)
- P.166, next-to-last line: Replace “*nonempty subset S of R*” by “*subset S of R containing 0 and 1*”.
- P.167: Replace the top four lines by:
2. A subfield of a field F is a subring S of F such that if $a \in S$ and $a \neq 0$, then $a^{-1} \in S$.
3. An ideal of a ring R is a subset I of R such that $0 \in I$, and if $a, b \in I$ then $a+b \in I$ and $a-b \in I$, and if $a \in I$ and $r \in R$, then ar and ra lie in I .
- P.167, first display: To the two equations shown, add a third: $\varphi(1) = 1$; and at the end of the display, change “for all $r \in R, s \in S$ ” to “for all $r, s \in R$ ”. (That the condition $\varphi(1) = 1$ was intended is shown by p.188, parenthetical note in Exercise 17.2.)
- P.167, Example 16.6, part 2, first line: $\mathbb{K}[t]$ should be $K[t]$.
- P.167, third-from-last line: “Its kernel is R ” should be “Its kernel is I ”.
- P.169, last paragraph of §16.3: This seems to refer to a different proof of Proposition 2.3 from the one actually given; namely, one that shows that a polynomial over \mathbb{C} of degree $\leq n$ which has all of $0, \dots, n$ as roots must be zero. (Key step: Since $f(n) = 0$, we can write $f(t) = (t-n)g(t)$, where g has degree $\leq n-1$ and has $0, \dots, n-1$ as roots, allowing us to complete the proof by induction.)
- P.169, next-to-last line: In place of “*Every prime subfield*” I recommend “*For every field K , the prime subfield of K* ”, as clearer.
- P.170, third line after Definition 16.10: Change “Exercise 16.2” to “Exercises 16.5 and 16.6”.
- P.171, line 2: “is an integer” should be “as an integer”.
- P.172, third line after middle display: “Exercise 16.3” should be Exercise 16.7.
- P.173, top line: “Exercise 16.4” should be Exercise 16.8.
- P.178, second paragraph of §17.2: The sentences beginning near the end of the first line (“Simple transcendental extensions...”) and near the middle of the fourth line (“As before...”) say the same thing in different words. I suggest crossing out the former, since the latter gives a bit more detail.
- P.179, paragraph before Theorem 17.3: Delete the parenthetical comment on 4th through 6th lines, “(This is the only place we use the fact that m is monic. ...)”, since he isn’t using that fact.
- P.183, second table: Upper left-hand corner should be marked ‘ \cdot ’ (as in the second table on p.174).
- P.184, second display: The term $(p-r)!$ should be in the denominator along with $r!$.
- P.184, beginning of second line above Example 17.16: “7.12” should be 17.12.
- P.185, end of top line: τ^2 should be $(-\tau)^5$.

- P.189, Exercise 17.10: Change “there exist” to “there may exist”.
- P.190, Exercise 17.14(a): Change this to “The minimal polynomial over a field K of any element of an algebraic extension of K is irreducible over K .”
- P.190, Exercise 17.14(b): Before “irreducible” add “monic”. (Without this change, this would be a trick question; with it, it tests one’s understanding of an important result. My guess is that it was intended in the latter way.)
- P.192, first line of proof of Lemma 18.4: β should be β_1 .
- P.197, end of 4th line of §18.4: After “every quintic” add “with coefficients in K ”.
- P.202, second display: The term $s_2 t^3$ should be $s_2 t$.
- P.227, third line of proof of Theorem 20.1: “Theorem 16.1” should be Theorem 6.1 (p.67).
- P.228, Theorem 20.2: “ $p = p^n$ ” should be $q = p^n$.
- P.228, proof of Theorem 20.2: The end-of-proof sign \square should occur one paragraph later than it does, after “Therefore, $|K| = q$ ”.
- P.229, second line of proof of Lemma 20.6: Change “Then G must possess elements g_j whose orders are” to “Then for each j , G must possess an element g_j whose order is”.
- P.232, first line of Exercise 20.5: GF should be \mathbb{GF} .
- P.239, fourth line below first display of §21.3: “ $\mathbb{Q}(\theta\zeta):\mathbb{Q}$ ” should be $\mathbb{Q}(\theta\zeta):\mathbb{Q}(\theta)$.
- P.251, Definition 2.21. Change this to:
Let G be a permutation group (i.e., a subgroup of the group of all permutations of a set S). We say that G is transitive (or transitive on S) if for all $s, t \in S$ there exists $\gamma \in G$ such that $\gamma(s) = t$.
If s_0 is any element of S , then to show that a group G of permutations of S is transitive it is enough to show that for all $s \in S$ there exists $\gamma \in G$ such that $\gamma(s_0) = s$, because then given two elements s and t there will exist $\gamma, \delta \in G$ such that $\gamma(s_0) = s$, $\delta(s_0) = t$, whence $(\delta\gamma^{-1})(s) = t$.
- P.252, second line of proof of Proposition 22.3: The “Proposition 4” referred to is Proposition 11.4 (on p.126). I’m not sure what “Theorem 4” should be; maybe Theorem 17.4 (p.179).
- P.256, statement of Theorem 22.7, point 3: At the beginning, add “If $\Delta(f) \neq 0$, then”, and after “the Galois group of f ” add “, regarded as a group of permutations of the zeros of f ,”. In the next-to-last line of the proof of this theorem, both occurrences of “ \subset ” should be “ \subseteq ”.
- P.257, line before second display: Before the final word “define”, insert “let x_1, \dots, x_n be independent indeterminates, let β be defined as above, and for every $\sigma \in \mathbb{S}_n$ ”.
- P.257, line before next display: $s \neq$ should be $\sigma \neq$.
- P.258, two-line display just below middle of page: At the beginning, γ should be γ_x . In the next line, both occurrences of $\gamma\alpha^{-1}$ should be γ_α^{-1} .
- P.259, Exercise 22.4: In the first phrase, “ δ ” should be “ δ or $-\delta$ ”. In the first entry of the third row of the first determinant, a_1^2 should be α_1^2 . In the very first entry of the second determinant, λ_n should be λ_0 .
- P.260, Exercise 22.6 top line: Change “is one of” to “is conjugate to one of”.
- P.260, Exercise 22.7, parts (c) and (e): change “ g splits” to “ g is reducible”. (This change could optionally be made in part (d); in that part either version is correct.)
- P.262, Definition 23.1, last sentence: After “in the obvious way” add “in terms of \leq ”.
- P.267, Exercise 23.8(g): Change “can be ordered” to “can be given an ordering making it an ordered field”.
- P.271, line after second display: For “the superscripts” read “the superscripts on f ”.
- P.272, final display: The last ‘)’ on the left-hand side should be ‘]’, and the first ‘(’ on the right-hand side should be ‘[’.