Challenge Exercise 1
MATH 1271/3071 – 2011
Due Date: Oct 21, 2011

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the problem correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name.

Problem 1. **[5pts]** In class we introduced five logical operators: $\wedge, \vee, \sim, \rightarrow, \leftrightarrow$. However, do we need all of these operators? For example:

$$p \rightarrow q \equiv \sim p \vee q.$$

Hence, any time we see an implication $\rightarrow$, we can replace it with a statement using only $\sim$ and $\vee$.

(a) Rewrite the following statement so that it only involves the operators $\vee$ and $\sim$:

$$(p \vee q) \rightarrow (p \rightarrow q)$$

(b) Explain why can rewrite the operators $\rightarrow$, and $\leftrightarrow$ using only the operators $\wedge, \vee$ and $\sim$.

(c) Can we do the reverse, i.e., can we write each operator $\wedge, \vee$ and $\sim$ using only the operators $\rightarrow$, $\leftrightarrow$?

(d) Is it possible to use only two operators?

Problem 2. **[5pts]** The **depth** of a circuit is defined by specifying that the depth of the initial input is 0, and if a gate has n different inputs at depths $d_1, \ldots, d_n$, respectively, then its outputs have depth equal to $\max\{d_1, \ldots, d_n\} + 1$. The depth of a circuit is the maximum depth of the gates in a circuit.

(a) What is the depth of the circuit of Exercise 17 of Section 3.4 on page 89.

(b) Compute the depth of a *full adder* (on page 87).