These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the problem correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name.

Problem 1. [5pts] Let A be any set, and let $f : A \to A$ be any function. We define a sequence of sets $\{A_n \mid n \in \mathbb{N}\}$ where

$$A_0 = A \text{ and } A_{n+1} = f(A_n) \text{ for all } n \geq 0.$$

Prove the following statements:

(a) For all $n \geq 0$, $A_{n+1} \subseteq A_n$.

(b) Let

$$A^* = \bigcap_{n \in \mathbb{N}} A_n.$$

Then $f(A^*) \subseteq A^*$.

Problem 2. [5pts] Consider a standard 8×8 chessboard. In chess, the king can move one position in any direction on this board.

    (a) **[2pts]** Suppose we decided to limit the king's moves to either moving one position up or one position to the right. Along how many different paths can a king be moved from the lower-left corner position to the upper right-corner position on the standard 8×8 chessboard?

    (b) **[3pts]** Again, suppose that the king can only move one position up and one position to the right. However, this time when the the king moves from the lower-left corner position to the upper right-corner position, we do not allow the king to move above the diagonal of the board. That is, the king must only travel on or below the diagonal of the board. Count the number of ways to travel between the two corners.