Challenge Exercise 1
MATH 1281 – 2009
Due Date: Sept. 25, 2009

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the problem correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name.

Problem 1. [5pts] In class we introduced six logical operators: $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$, and $\oplus$. However, do we need all of these operators? For example, on page 25 in Table 7 you can find the logical equivalence:

$$p \rightarrow q \equiv \neg p \vee q.$$

Hence, any time we see an implication $\rightarrow$, we can replace it with a statement using only $\neg$ and $\vee$.

(a) Rewrite the following statement so that it only involves the operators $\vee$ and $\neg$:

$$(p \vee q) \rightarrow (p \rightarrow q)$$

(b) Explain why can rewrite the operators $\rightarrow$, $\leftrightarrow$ and $\oplus$ using only the operators $\wedge, \vee$ and $\neg$.

(c) Can we do the reverse, i.e., can we write each operator $\wedge, \vee$ and $\neg$ using only the operators $\rightarrow$, $\leftrightarrow$, and $\oplus$?

(d) Is it possible to use only two operators?

Problem 2. [5pts] Let $p(x)$ and $q(x)$ be propositional functions in the variable x with a given universe.

(a) Explain why if $\forall x\ p(x) \vee \forall x\ q(x)$ is true, then the statement $\forall x(p(x) \vee q(x))$ is true.

(b) Show that the converse of (a) is false by finding a counterexample (i.e., you need to pick a universe and propositional functions $p(x)$ and $q(x)$ such that the statement "If $\forall x(p(x) \vee q(x))$ is true, then $\forall x\ p(x) \vee \forall x\ q(x)$ is true" is a false statement).