Challenge Exercise 2
MATH 1281 – 2010
Due Date: Nov 12, 2009

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the problem correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name.

Problem 1. [**5pts**] Let A be any set, and let $f : A \to A$ be any function. We define a sequence of sets $\{A_n \mid n \in \mathbb{N}\}$ where

$$A_0 = A \text{ and } A_{n+1} = f(A_n) \text{ for all } n \geq 0.$$

Prove the following statements:

(a) For all $n \geq 0$, $A_{n+1} \subseteq A_n$.

(b) Let

$$A^* = \bigcap_{n \in \mathbb{N}} A_n.$$

Then $f(A^*) \subseteq A^*$.

Problem 2. [**5pts**] Let $f : A \to B$ be any function. For any subset $C \subseteq B$, define:

$$f^{-1}(C) = \{a \in A \mid f(a) \in C\}.$$

That is, $f^{-1}(C)$ contains all the elements of A that get mapped into C.

Let $D \subseteq A$ and $C \subseteq B$. Show:

(i) $D \subseteq f^{-1}(f(D))$
(ii) $f(f^{-1}(C)) \subseteq C$