MATH 1ZC3/1B03: Test 2 - Version 1
Instructors: Hildum, Lozinski, Tam
Date: March 25, 2014 - Group A
Duration: 90 min.

Name: ____ **ID #:** ________________

Instructions:

This test paper contains 20 multiple choice questions printed on both sides of the page. The questions are on pages 2 through 13. Pages 14 to 16 are available for rough work. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCIES TO THE ATTENTION OF THE INVIGILATOR.

Select the one correct answer to each question and ENTER THAT ANSWER INTO THE SCAN CARD PROVIDED USING AN HB PENCIL. Room for rough work has been provided in this question booklet. You are required to submit this booklet along with your answer sheet. HOWEVER, NO MARKS WILL BE GIVEN FOR THE WORK IN THIS BOOKLET. Only the answers on the scan card count for credit. Each question is worth 1 mark. The test is graded out of 20. There is no penalty for incorrect answers.

Computer Card Instructions:

IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED. YOUR TEST RESULTS DEPEND UPON PROPER ATTENTION TO THESE INSTRUCTIONS.

The scanner that will read the answer sheets senses areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid.

- Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet **MUST** be signed in the space marked SIGNATURE.
- Mark your student number in the space provided on the sheet on Side 1 **and fill the corresponding bubbles underneath**.
- Mark only **ONE** choice (A, B, C, D, E) for each question.
- Begin answering questions using the first set of bubbles, marked “1”.

1. Express the complex number
$$10 - 10i$$
in polar form.
   (a) $10\sqrt{2}e^{(\pi/4)i}$
   (b) $5\sqrt{2}e^{(-\pi/2)i}$
   (c) $5\sqrt{2}e^{(3\pi/4)i}$
   (d) $\frac{5}{\sqrt{2}}e^{(-3\pi/2)i}$
   (e) $10\sqrt{2}e^{(-\pi/4)i}$

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2. Solve for z if $\bar{z}(1+i) + (-1+i) = 3 - 2i$.
   (a) $\frac{7}{2} + 2i$
   (b) $\frac{1}{2} + \frac{7}{2}i$
   (c) $7 - 2i$
   (d) $4 - 3i$
   (e) $3 + 4i$

3. Which of the following is NOT an argument (between 0 to 2π) of a root of $z^4 = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$?

   (a) $\frac{\pi}{10}$

   (b) $\frac{3\pi}{5}$

   (c) $\frac{7\pi}{10}$

   (d) $\frac{11\pi}{10}$

   (e) $\frac{8\pi}{5}$

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4. Which of the following complex numbers lies on the unit circle (a circle of radius 1)?

   (a) $1+i$

   (b) $\frac{1}{3}+\frac{2}{3}i$

   (c) $\frac{3}{5}+\frac{4}{5}i$

   (d) $\frac{4}{7}+\frac{5}{7}i$

   (e) $\frac{5}{9}+\frac{7}{9}i$

5. Let $\omega = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$. Which of the following is true?
(i) $\bar{\omega}^2 = \omega^4$.
(ii) $\omega^2 = -\omega^4$.
(iii) ω is one of the cube roots of 1.

(a) (iii) only.
(b) (ii) and (iii) only.
(c) (i) only.
(d) (i) and (iii) only.
(e) (i), (ii), and (iii).

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6. If $z_1 = 2e^{(\pi/9)i}$ and $z_2 = 10e^{(\pi/3)i}$, what is $(z_1)^3(z_2)^2$?

(a) $800(1+i)$
(b) $400\sqrt{3} + 400i$
(c) $800\pi/81$
(d) $400 - i400\sqrt{3}$
(e) -800

7. Let
$$A = \begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix}.$$
What is the eigenvalue associated with the eigenvector
$$\begin{bmatrix} 1 + i\sqrt{3} \\ -2 \end{bmatrix}?$$

(a) $2 + i\sqrt{3}$

(b) $\sqrt{3}/2 + i/2$

(c) $1/2 + i\sqrt{3}/2$

(d) $\sqrt{3} + i2$

(e) $1 + i\sqrt{3}$

8. Given two vectors in n-space, $\mathbf{v}$ and $\mathbf{w}$ such that $\mathbf{v} \cdot \mathbf{w} = 0$, which of the following *must* be true?

(i) either $\mathbf{v} = 0$ or $\mathbf{w} = 0$
(ii) $\mathbf{v}$ and $\mathbf{w}$ are orthogonal vectors
(iii) $\text{proj}_{\mathbf{v}}\mathbf{w} = \mathbf{0}$

(a) (i) and (ii) only
(b) (ii) and (iii) only
(c) (i), (ii) and (iii)
(d) (i) and (iii) only
(e) (ii) only

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9. Find $\mathbf{v} \cdot \mathbf{w}$ if $\mathbf{v} = -\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} - \mathbf{k}$.

(a) $\mathbf{v} \cdot \mathbf{w} = -4$
(b) $\mathbf{v} \cdot \mathbf{w} = 0$
(c) $\mathbf{v} \cdot \mathbf{w} = -3$
(d) $\mathbf{v} \cdot \mathbf{w} = -2\mathbf{i} - \mathbf{k}$
(e) $\mathbf{v} \cdot \mathbf{w} = \mathbf{i} - 2\mathbf{j}$

10. Let $A(1,-1,1)$, $B(0,3,2)$ and $C(-1,-2,0)$ be points in $\mathbf{R}^3$. Find the cosine of the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(a) $\cos(\theta) = -\dfrac{1}{2\sqrt{3}}$

(b) $\cos(\theta) = -\dfrac{3}{\sqrt{3}}$

(c) $\cos(\theta) = 0$

(d) $\cos(\theta) = -\dfrac{3}{\sqrt{6}}$

(e) $\cos(\theta) = \dfrac{1}{3\sqrt{6}}$

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11. Find $\mathbf{v} \times \mathbf{w}$ if $\mathbf{v} = -\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} - \mathbf{k}$.

(a) $\mathbf{v} \times \mathbf{w} = -\mathbf{i} - \mathbf{k}$

(b) $\mathbf{v} \times \mathbf{w} = -\mathbf{i} - 2\mathbf{j} - \mathbf{k}$

(c) $\mathbf{v} \times \mathbf{w} = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(d) $\mathbf{v} \times \mathbf{w} = \mathbf{i} - 2\mathbf{j}$

(e) $\mathbf{v} \times \mathbf{w} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$

12. Let $\mathbf{v} = -\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} - \mathbf{k}$. Find the vector component of $\mathbf{w}$ along $\mathbf{v}$ and the component of $\mathbf{w}$ orthogonal to $\mathbf{v}$.

   (a) $(1,\ 2,\ -1)$ and $(-1,\ -2,\ 1)$
   (b) $(2,\ 4,\ -2)$ and $(0,\ -4,\ -1)$
   (c) $(1,\ 2,\ -1)$ and $(1,\ -2,\ 0)$
   (d) $(1/2,\ 1,\ -1/2)$ and $(3/2,\ -1,\ -1/2)$
   (e) $(1/2,\ 1,\ -1/2)$ and $(-1/2,\ 0,\ 1/2)$

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13. Find the equation of the plane that passes through the point $P(2, 0, 3)$ parallel to the vectors $\mathbf{v} = (-1, 0, 4)$ and $\mathbf{w} = (2, -2, 4)$.

   (a) $-2x + 16z = 44$
   (b) $2x - 2y + 4z = 16$
   (c) $-x + 4z = 10$
   (d) $x - 2y + 8z = 26$
   (e) $8x + 12y + 2z = 22$

14. Find the distance between the 2 parallel planes $3x-4y+z=2$ and $6x-8y+2z=0$.

    (a) $\sqrt{3}/12$
    (b) $4/\sqrt{13}$
    (c) $2/\sqrt{26}$
    (d) $\sqrt{2}/12$
    (e) $18/\sqrt{2}$

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15. Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be parallel vectors, and $\mathbf{v}_3$ be a vector orthogonal to $\mathbf{v}_1$ and $\mathbf{v}_2$. All three vectors are non-zero. Which of the following *must* be true, where t_1, t_2, and t_3 are scalars.

    (i) $\mathbf{x} = t_1\mathbf{v}_1 + t_2\mathbf{v}_2$ defines the equation of a plane in $\mathbf{R}^n$
    (ii) $\mathbf{x} = t_1\mathbf{v}_2 + t_2\mathbf{v}_3$ defines the equation of a plane in $\mathbf{R}^n$
    (iii) There exists a plane in $\mathbf{R}^n$ containing all three vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

    (a) (i) and (ii) only
    (b) (ii) and (iii) only
    (c) (i) and (iii) only
    (d) (ii) only
    (e) (iii) only

16. At time t, at an amusement park, the fraction of guests riding rides (r_t), eating food (f_t), and watching shows (s_t) is given by a state vector $\mathbf{v_t} = [r_t, f_t, s_t]^T$. The state vector at any later point in time t is determined by a discrete time dynamical system driven by the matrix A where

$$A = \begin{bmatrix} 0.9 & 0.2 & 0 \\ 0 & 0.8 & 0.6 \\ 0.1 & 0 & 0.4 \end{bmatrix}$$

In the long run steady state, what fraction of guests are watching a show?

(a) 1/5
(b) 1/10
(c) 1/3
(d) 1/8
(e) 1/6

17. A city bus is either driving a route, out of service, or parked outside a Tim Hortons. A bus that is driving a route one hour has a $1/5$ probability of being out of service the next hour, and a $1/5$ probability of being at a Tim's (otherwise it is still driving a route). If a bus is out of service, the next hour it will NOT be at a Tim's, but will either be driving a route, or it will still be out of service, with equal probability. A bus parked outside a Tim's will be driving a route the next hour. If you are presently sitting on a bus, riding a route, what is the probability that your bus will be parked outside a Tim Hortons two hours from now?

   (a) $1/5$
   (b) $1/3$
   (c) $1/10$
   (d) $3/5$
   (e) $3/25$

18. Which of the following series of Matlab commands will NOT produce the following matrix: $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

   (a) `A = eye(3); A(1,2) = 2; A(2,3) = 3`
   (b) `x = [1 1 1]; y = [2 3];  A = diag(x,0); A = diag(y,1)`
   (c) `x = [2 3]; A = diag(x,1); eye(3)+A`
   (d) `A = [1 2 0; 0 1 3; 0 0 1]`
   (e) `eye(3) + diag([2 3],1)`

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19. Which of the following sets are closed under addition?
(suggestion: consider drawing some of these sets on the complex plane)

(i) The set of complex numbers.
(ii) The set of complex numbers with modulus (absolute value) 1.
(iii)The set of complex numbers z such that either $z = 0$, or $\arg(z) = \frac{\pi}{2}$, or $\arg(z) = -\frac{\pi}{2}$.

   (a) None of the above.
   (b) (i) and (ii) only.
   (c) (i) and (iii) only.
   (d) (ii) and (iii) only.
   (e) (i), (ii), and (iii).

20. Let V be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operations on $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$:

$$\mathbf{u} \oplus \mathbf{v} = (u_1 v_1, u_2 v_2)$$

$$k \odot \mathbf{u} = (ku_1, ku_2)$$

Which of the following properties is NOT satisfied?

(a) V is closed under vector addition

(b) V is closed under scalar multiplication

(c) V has a zero vector, and it is (1,1)

(d) $k \odot (\mathbf{u} \oplus \mathbf{v}) = (k \odot \mathbf{u}) \oplus (k \odot \mathbf{v})$

(e) $1 \odot \mathbf{u} = \mathbf{u}$

END OF TEST QUESTIONS

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