
Challenge Exercise 3
MATH 2275 – Winter 2012
Due Date: March 8, 2012

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the solutions correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name. For those of you in Math 2232/2234 (Abstract Algebra), you are encouraged to write your solutions as a formal proof.

Problem. Let \mathbf{u} be any unit vector in \mathbb{R}^n . We define an $n \times n$ matrix Q by

$$Q = I_n - 2\mathbf{u}\mathbf{u}^T.$$

The matrix Q is called a **Householder matrix**.

For the following questions, assume $\mathbf{u} \in \mathbb{R}^2$. So the Householder matrix is a 2×2 matrix.

- (a) [2pts] Show that Q is a symmetric matrix (a matrix A is symmetric if $A^T = A$).
- (b) [4pts] Show that Q is an orthogonal matrix (see the definition on page 344).
- (c) [2pts] Show that $Q^2 = I_2$.
- (d) [2pts] Show that

$$Q\mathbf{v} = \begin{cases} -\mathbf{v} & \text{if } \mathbf{v} \in \text{Span}\{\mathbf{u}\} \\ \mathbf{v} & \text{if } \mathbf{v} \cdot \mathbf{u} = 0 \end{cases}$$