Math 2R03 Final Exam Info Sheet

The purpose of this handout is to help you study by listing the concepts, definitions, and results you will need to know for the final exam.

Final Exam Information. The final will be on Thursday, April 29, 2021. The final will be two hours and 30 minutes long from 7:30PM to 10:00PM. The test will be given via CrowdMark. I will release the questions 10 minutes before the start time at 7:20PM, and I will give you 20 extra minutes (until 10:20PM) to upload your questions. I will be available online during that time via zoom to answer any questions.

The midterm will be open book. You can use your textbook, notes, class videos and notes, and Octave/Matlab. You may not use the help of any other person or website. There are 13 questions. You will have to answer all of Questions 1-6. You will have to do 5 of Questions 7-12. Question 13 is a bonus questions.

Material Covered. The final exam is cumulative, although there will be more of an emphasis on the material since the last midterm. For a breakdown on what you need from Lectures 1-23, see the review sheets for Midterms 1 and 2. Below is a breakdown of what you will need to know from Lecture 24-36. Note that when you are learning definitions, it is good to know an example of that definition, and an example of an object that does not satisfy the definition.

Section 8.C Know what the characteristic polynomial and minimal polynomial of an operator is. Know Theorem 8.36, the Cayley-Hamilton theorem, Theorem 8.46, 8.49.

Section 8.D Know the definition of a Jordan Basis of an operator. Also know Theorem 8.60. Also know Theorem 8.55 on the basis of a nilpotent operator. (You will not be asked to find the Jordan basis of an operator.)

Section 6.A Know the definition of an inner product (6.3) and be able to determine if a given function is an inner product. Know what an inner product space is, and know the properties of an inner product (e.g. Theorem 6.7). Know what we mean by a norm, and orthogonal vectors. Know the Pythagorean Theorem, Cauchy-Schwarz inequality, the triangle inequaltiy, and the parallelogram equality.

Section 6.B Know what it means for a collection of vectors to be orthonormal, and know what an orthonormal basis is. Important properties of orthonormal sets are in Theorems 6.25, 6.26, 6.28, and 6.30. Know how to apply the Gram-Schmidt procedure. Know 6.34, 6.35, and Schur's Theorem. Know what we mean by a linear functional, and know the Riesz Representation Theorem. In particular, be able to find the vector u in this theorem.

Section 7.A Know what an adjoint of an operator is. Know the properties of the adjoint (Theorem 7.6). You do not need to know Theorem 7.7. Know what we mean by the conjugate transpose, and how to find the matrix of the adjoint. Know what a self-adjoint operator is, and know some of its properties (e.g Theorem 7.13). You will not be tested on 7.14, 7.15, or 7.16. Know what a normal operator is and some of it's properties (e.g., 7.20, 7.21, and 7.22).

Section 7.B Know the Complex Spectral Theorem and the Real Spectral Theorem. For both theorems, you need to know how to apply the statements. You will not be tested on their proofs.

Section 7.C Know what a positive operator is, and how to check if an operator is positive (e.g., use Theorem 7.35). Know want an isometry is, and how to check if an operator is an isometry (e.g. use Theorem 7.42). Know the properties of both operators.

Section 7.D Know the statements of both Polar Decomposition and Singular Value Decomposition, and how to apply them. Know what we mean by the singular values, and how to find them (e.g., Theorem 7.52).

If you have questions, please feel free to email me. The TA and I will also schedule a review session and office hours during the exam session. Please see Avenue-to-Learn for more details. Good luck!