MATH 2R03 (LINEAR ALGEBRA II)
DUE: JANUARY 23, 2021

HOMEWORK ASSIGNMENT 1

All of the questions from Part A will be graded. One of the questions from Part B will be graded in detail, while the other will be marked for completion. Assignments will be submitted via *Crowdmark*. You will be graded on your solution *and* how well you write your proof.

Part A. [Short Questions; 4pts]

Exercise 1. Let V be a vector space. Suppose $v, w \in V$ satisfy $v + w = v$. Prove that $w = 0$.

Exercise 2. Consider the two subspaces $U = \{(3x_1, x_1) \mid x_1 \in \mathbb{R}\}$ and $W = \{(-x_1, 3x_1) \mid x_1 \in \mathbb{R}\}$ of $\mathbb{R}^2$. Prove that $U + W$ is a direct sum.

Part B. [Proof Questions; 6pts]

Exercise 3. Let U_1, U_2 be two subspaces of a vector space V. Prove that $U_1 \cap U_2$ is also a subspace of V. Given an example to show that $U_1 \cup U_2$ is not necessarily a subspace.

Exercise 4. Fix a non-zero real number $a \in \mathbb{R}$. Suppose that

$$U = \{(ax_1, x_1) \mid x_1 \in \mathbb{R}\}.$$

Find a subspace W of $\mathbb{R}^2$ such that $\mathbb{R}^2 = U \oplus W$.

Hint. Make sure you prove that your set W is a subspace and $U + W = \mathbb{R}^2$. Exercise 2 may also help.