Math 2R03 (Linear Algebra II)
Due: January 30, 2021

Homework Assignment 2

All of the questions from Part A will be graded. One of the questions from Part B will be graded in detail, while the other will be marked for completion. Assignments will be submitted via *Crowdmark*. You will be graded on your solution *and* how well you write your proof.

Part A. [Short Questions; 4pts]

Exercise 1. Let $U = \{(x_1, x_2, x_3, x_4) \in F^4 \mid 2x_1 = x_2, 3x_3 + 6x_4 = 0\}$. This is a subspace of F^4 (you may wish to convince yourself of this fact, but you don't need to prove it). Prove that $(\frac{1}{2}, 1, 0, 0), (0, 0, -2, 1)$ is a basis for U (make sure to prove that your set of vectors is linearly independent and spans U).

Exercise 2. Suppose that v_1, v_2, v_3 are linearly independent in V. Is the set $v_1 + v_2, 2v_2, v_3 + v_2$ also linearly independent in V? Justify your answer.

Part B. [Proof Questions; 6pts]

Exercise 3. Let $v_1, \ldots, v_n$ and $w_1, \ldots, w_m$ be vectors in vector space V. Prove that

$$\operatorname{span}(v_1, \ldots, v_n) = \operatorname{span}(w_1, \ldots, w_m)$$

if and only if $v_i \in \operatorname{span}(w_1, \ldots, w_m)$ for all $i = 1, \ldots, n$ and $w_j \in \operatorname{span}(v_1, \ldots, v_n)$ for all $j = 1, \ldots, m$.

Remark. This result is very useful since it gives us a method to prove two sets of elements produce the same spanning set.

Exercise 4. Let $V = \mathcal{P}_4(\mathbb{R})$ be the vector space of polynomials of degree 4 or less, and suppose that the polynomials $q_1, q_2, q_3, q_4, q_5 \in V$ have the property that $q_1(2021) = q_2(2021) = q_3(2021) = q_4(2021) = q_5(2021) = 0$. Show that the polynomials $q_1, \ldots, q_5$ are linearly dependent in V.

Hint. You can use the fact fact that $q(2021) = 0$ implies $q(z) = (z - 2021)q'(z)$ where $\deg q'(z) = \deg q(z) - 1$ without proof.