MATH 2R03 (LINEAR ALGEBRA II)
DUE: MARCH 13, 2021

HOMEWORK ASSIGNMENT 6

All of the questions from Part A will be graded. One of the questions from Part B will be graded in detail, while the others will be marked for completion. Assignments will be submitted via *Crowdmark*. You will be graded on your solution *and* how well you write your proof.

Part A. [Short Questions; 4pts]

Exercise 1. Let $T \in \mathcal{L}(\mathbb{C}^2)$ be the linear operator given by

$$T(x, y) = (50x - 20y, 125x - 50y).$$

Prove that T is a nilpotent operator.

Exercise 2. Let $T \in \mathcal{L}(\mathbb{C}^2)$ be the same linear operator as in the previous question. Find a basis for $\mathbb{C}^2$ such that the matrix $\mathcal{M}(T)$ with respect to this basis has the form described in Theorem 8.19.

Part B. [Proof Questions; 6pts]

Exercise 3. Let $T \in \mathcal{L}(\mathbb{C}^2)$ be an operator with just one eigenvalue λ. Show that $T - \lambda I \in \mathcal{L}(\mathbb{C}^2)$ is nilpotent.

Hint. Use Theorems 5.27 and 5.32 to show that there is a basis v_1, v_2 of $\mathbb{C}^2$ such that

$$\mathcal{M}(T) = \begin{bmatrix} \lambda & \delta \\ 0 & \lambda \end{bmatrix}$$

with respect to this basis. Here δ is some complex number.

Exercise 4. Consider $T \in \mathcal{L}(V)$ with $\dim V = n$. Suppose there is a subspace $H \subseteq V$ with $\dim H = n - 1$ and a vector v not in H such that

$$T(h) = h \text{ for all } h \in H$$

and

$$T(v) = -v.$$

Show that T is diagonalizable. In particular, show that there is a basis of V such that

$$\mathcal{M}(T) = \begin{bmatrix} -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \vdots & \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Remark. Note that H is an invariant under T.