MATH 2R03 (LINEAR ALGEBRA II)
DUE: APRIL 13, 2021 (NOTE DATE CHANGE!)

HOMEWORK ASSIGNMENT 9

All of the questions from Part A will be graded. One of the questions from Part B will be graded in detail, while the others will be marked for completion. Assignments will be submitted via *Crowdmark*. You will be graded on your solution *and* how well you write your proof.

Part A. [Short Questions; 6pts]

Exercise 1. Consider the operator $T \in \mathcal{L}(\mathbb{C}^2)$ given by

$$T(x, y) = (-x + 2y, -2x + y).$$

Find the polar decomposition of T.

Remark. To answer this question, turn this into a question about matrices and use the approach outlined at the end of Lecture 35. In particular, find a positive definite matrix R such that $R^2 = A^T A$ and an orthogonal matrix U where $A = UR$ where $A = \mathcal{M}(T)$.

Exercise 2. With same operator T as in Exercise 1, find the singular value decomposition of T.

Remark. The question is asking you to find two singular values s_1, s_2, and to find two orthonormal bases e_1, e_2 and f_1, f_2 of $\mathbb{C}^2$ so that

$$Tv = s_1\langle v, e_1\rangle f_1 + s_2\langle v, e_2\rangle f_2$$

for all v. Look at Lecture 36 and use Exercise 1.

Part B. [Proof Questions; 6pts]

Exercise 3. Let V be a finite dimensional vector space, and let $T \in \mathcal{L}(V)$. Prove that $T + T^*$ is a self-adjoint operator. What does this statement mean for $n \times n$ matrices with entries in $\mathbb{R}$?

Exercise 4. Let V be a finite dimensional complex vector space. Suppose that $E \in \mathcal{L}(V)$ satisfies $E^2 = E$. Show that E is self-adjoint if and only if E is normal.

Hint. The direction ($\Rightarrow$) doesn't require $E^2 = E$. For the reverse direction, to show $E = E^*$, show that there is a basis $v_1, \ldots, v_n$ such that $Ev_i = E^*v_i$ for all i (do you see why this enough?). Use the fact that $E^2 = E$ to describe the possible eigenvalues of E (you may want to use results from Section 8.C).