Math 2R03 Midterm 2 Info Sheet

The purpose of this handout is to help you study by listing the concepts, definitions, and results you will need to know for the midterm.

Midterm Information. The midterm will be on Thursday, March 18, 2021 The midterm will be 50 minutes long (during regular class time 12:30-1:20). The test will be given via CrowdMark. I will release the questions 10 minutes before the start time at 12:20PM, and I will give you 25 extra minutes (until 1:45PM) to upload your questions. I will be available online during that time via zoom to answer any questions.

The midterm will be open book. You can use your textbook, notes, class videos and notes, and Octave/Matlab. You may not use the help of any other person or website. There are 8 questions (5 short answer and 3 longer questions). The midterm is out of 25 points.

Material Covered. The midterm will cover the material we discussed in class in Lectures 12-23 (Chapters 3.D, 4, 5.A-5.C, 8.A-8.B). Below is a breakdown of what you will need to know from each section. Note that when you are learning definitions, it is good to know an example of that definition, and an example of an object that does not satisfy the definition.

Section 3.C Know what it means for a map to be invertible, and what it means for a linear map to have an inverse. Know Theorem 3.56 which gives an equivalent way to check invertibility. Know what it means for two vector spaces to be isomorphic, and know what an isomorphism is. Know Theorem 3.59, 3.60, and 3.61. Know what we mean by the matrix of a vector (Definition 3.62), and Theorem 3.65. Know what we mean by an operator, and know Theorem 3.69 which gives equivalent ways to check if an operator is invertible.

Chapter 4 For Chapter 4, you only need to know what a root of a polynomial is, how to factorize a polynomial over $\mathbb{C}$ (Theorem 4.14) and how to factorize a polynomial over $\mathbb{R}$ (Theorem 4.17). You should also know the Fundamental Theorem of Algebra. You will not be tested on the proofs of these results.

Section 5.A Know what an invariant subspace is. Know what an eigenvalue and and eigenvector of a linear operator are. Know the equivalent conditions for λ to be an eigenvalue (Theorem 5.6). Know Theorem 5.10 and 5.11. You do not need to know the material on page 137.

Section 5.B Know what it means to apply a polynomial to an operator. Know Theorem 5.21 (pay attention to the hypotheses). Know what we mean by $\mathcal{M}(T)$ for an operator $T \in \mathcal{L}(V)$. Know what it meant by an upper-triangular and diagonal matrix. Know the conditions for being an upper-triangular matrix, and know Theorem 5.27. Also know Theorem 5.30 and 5.32.

Section 5.C Know what we mean by the eigenspace of an operator T with eigenvalue λ. Know what it means for an operator to be diagonalizable, and know Theorem 5.41 which gives equivalent conditions for an operator to be diagonalizable. Know Theorem 5.44.

Section 8.A Know Theorems 8.2, 8.3, 8.4, and 8.5 which all describe how the null space of T^i changes as i increases. Know what is meant by a generalized eigenvector and generalized eigenspace. Know Theorem 8.11 and 8.13. Know what is meant by a nilpotent operator and Theorems 8.18 and 8.19.

Section 8.B Know Theorem 8.21 and how to use it to decompose a vector space into generalized eigenspaces. Know Theorem 8.23. Know what is meant by multiplicity, and the connection to algebraic multiplicity and geometric multiplicity. Know what is meant by a block diagonal matrix and know Theorem 8.29. You do not need to know the material on square roots (pages 258-259).

If you have questions, please feel free to email me. Good luck!