MATH 2R03 (THEORY OF LINEAR ALGEBRA)
DUE: JANUARY 22, 2022

HOMEWORK ASSIGNMENT 1

All of the questions from Part A will be graded. One of the questions from Part B will be graded in detail, while the other will be marked for completion. Assignments will be submitted via *Crowdmark*. You will be graded on your solution *and* how well you write your proof.

Part A. [Short Questions; 4pts]

Exercise 1. Let V be a vector space over a field F. Show that for all $x \in V$ there is a unique y such that $x + 2022y = 0$.

Exercise 2. Suppose that U and W are subspaces of a vector space V. Prove that if $W \subseteq U$, then $U + W = U$.

Part B. [Proof Questions; 6pts]

Exercise 3. Let $V = F^\infty$ and let

$$W = \{(x_1, \dots, x_{2021}, 0, x_{2023}, x_{2024}, \dots) \mid x_i \in F\} \subseteq V.$$

That is, W consists of all the elements of F^∞ whose 2022^{nd} coordinate is zero. Prove that W is a subspace of V.

Exercise 4. Consider the subspace $U = \{(x, y, 2022x) \mid x, y \in \mathbb{R}\} \subseteq \mathbb{R}^3$. Find a subspace $W \subseteq \mathbb{R}^3$ such that

$$\mathbb{R}^3 = U \oplus W.$$

Hint. Make sure you prove that your set W is a subspace and $U + W = \mathbb{R}^3$.

Additional Suggested Problems. [Not graded]

Problems 1.A # 11, 1.B # 1, 2, 1.C # 6, 10