Math 2R03 (Theory of Linear Algebra)
Due: April 12, 2022

Homework Assignment 9

All of the questions from Part A will be graded. One of the questions from Part B will be graded in detail, while the others will be marked for completion. Assignments will be submitted via *Crowdmark*. You will be graded on your solution *and* how well you write your proof.

Part A. [Short Questions; 4pts]

Exercise 1. Consider the operator $T \in \mathcal{L}(\mathbb{C}^3)$ given by

$$T(x, y, z) = (x + y, y + z, x + cz)$$

where c is an unknown constant. Find the value of c so that T is a normal operator.

Exercise 2. Consider the operator $T \in \mathcal{L}(\mathbb{C}^2)$ given by

$$T(x, y) = (x - iy, ix + y).$$

Is the operator T positive? Is the operator T an isometry? Justify each answer.

Part B. [Proof Questions; 6pts]

Exercise 3. Let V be a finite dimensional vector space over F. Let $T \in \mathcal{L}(V)$ be an operator such that $T^\star = -T$ (these operators are sometimes called *skew-adjoint*). Prove that if λ is an eigenvalue of T, then $\lambda = 0 + bi$ for some $b \in \mathbb{R}$., i.e., λ has no real part.

Hint. Mimic the proof of Theorem 7.13.

Exercise 4. Let $V = \mathbb{C}^4$. Suppose that $T \in \mathcal{L}(V)$ has only two distinct eigenvalues, $\lambda_1 = 2022$ and $\lambda = -1$. Further, suppose that the minimal polynomial of T is $p(z) = (z - 2022)(z + 1)^2$. Prove that T is **not** normal.

Additional Suggested Problems. [Not graded]

Problems 7.A # 1, 2 (hint: prove contrapositive) 6, 7 7.B # 1, 4, 5, 6 7.C # 6, 7, 10 7.D # 4, 5, 15