Math 3GR3 (Abstract Algebra)
Due: September 20, 2018

Homework Assignment 1

Do all of the questions. Three to four questions will be graded in detail (five points each), while the remaining questions will be graded for completion (one point each).

Exercise 1. Let $A = \{1,2,3,4\}$, $B = \{1,2,3\}$ and $C = \{2,4\}$. Determine the elements of the following sets:

(*a*) $B \times C$
(*b*) $C \times B$
(*c*) $A \setminus B$
(*d*) $(A \cap B) \cup C$.

Exercise 2. Let A, B, and C be sets. Prove that

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

Exercise 3. Let A, B, and C be sets, and let $f : A \to B$ and $g : B \to C$ be functions. Suppose that the function $(g \circ f) : A \to C$, the composition of f and g, is a surjective function. Prove that the function $g : B \to C$ is also a surjective function.

Exercise 4. Consider the set

$$R = \{(x,y) \mid x^2 = y^2\} \subseteq \mathbb{Z} \times \mathbb{Z}.$$

(*a*) Prove that R is an equivalence relation on the set $\mathbb{Z}$.
(*b*) Describe the equivalence classes of R.

Exercise 5. Is the set $R = \{(x,y) \mid x \leq y\} \subseteq \mathbb{Z} \times \mathbb{Z}$ an equivalence relation?

Exercise 6. Use induction to prove that 3 divides $n^3 - n$ for all $n \geq 1$.

Exercise 7. Let a and b be non-negative integers, and suppose that there exists integers r and s such that $ar + bs = 1$. Show that $\gcd(a,b) = 1$.

Now give an example to show that this fact cannot be generalized. That is, show that the following statement is false: if there exists integers r and s such that $ar + bs = t > 1$, then $\gcd(a,b) = t$.

Remark. The purpose of the above exercise is to understand when the converse of Corollary 2.11 (page 26) holds.

Exercise 8. Go to `http://abstract.ups.edu/aata/aata.html` and do the SAGE tutorials for Chapters 1 and 2. Then find $\gcd(123456789, 934127856)$ and the two integers r and s such that $123456789r + 934127856s = \gcd(123456789, 934127856)$.