Math 3GR3 (Abstract Algebra)
Due: October 4, 2018

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## Homework Assignment 2

Do all of the questions. Three to four questions will be graded in detail (five points each), while the remaining questions will be graded for completion (one point each).

**Exercise 1**. Consider a rectangle that is not square (i.e., the four sides do not all have the same length). Describe all the symmetries of the rectangle. Write down the Cayley table for the group of symmetries.

**Exercise 2**. Find all the integers $k$ that make $4 \equiv 2k \pmod 7$ true.

**Exercise 3**. Write out all the elements of $U(10)$. Then, for each element of $U(10)$, compute its order.

**Exercise 4**. Prove: If $G$ is a group such that $g^2 = e$ for all $g \in G$, then $G$ is abelian. Then give an example to show that the converse statement is false.

**Exercise 5**. Let $G$ be a group, and suppose that $a, b \in G$ satisfy $(ab)^n = e$ for some integer $n \geq 2$. Prove that $(ba)^n = e$.

*Remark.* If $G$ is an abelian group, then this is trivial to prove since $ab = ba$. However, you cannot assume $G$ is abelian in this exercise.

**Exercise 6**. Let $G = GL_2(\mathbb{R})$, i.e., the group of $2 \times 2$ invertible matrices. Prove that the set

$$H = \{A \in GL_2(\mathbb{R}) \mid \det(A) = 1\}$$

is a subgroup of $G$.

**Exercise 7**. Consider the multiplicative group $G = \mathbb{C}^*$. Prove that if $z = a+bi \in G$ and $a^2+b^2 \neq 1$, then the order of $z$ in $G$ is infinite.

*Remark.* The contrapositve statement tells us that if $z \in G$ has finite order, then $a^2 + b^2 = 1$, i.e., the only elements of finite order in $\mathbb{C}^\star$ must be in the circle group.

**Exercise 8**. Go to `http://abstract.ups.edu/aata/aata.html` and do the SAGE tutorials for Chapters 3 and 4. Consider the multiplicative groups $U(33), U(34)$, and $U(35)$. Use SAGE to determine which groups are cyclic. Provide your SAGE code to justify your answer.

*Hint.* Try to adapt the code on this page `http://abstract.ups.edu/aata/cyclic-sage-exercises.html` to the problem.

*Remark.* Gauss proved exactly when $U(n)$ is a cyclic group. See if you can find this result to double check your answer!