Math 3GR3 Midterm 1 Info Sheet

The purpose of this handout is to help you study by listing the concepts, definitions, and results you will need to know for the midterm.

Midterm Information. The midterm will be on Thursday, October 18, 2018 at 3:30PM. The midterm will take place in

T13 123

and will be 50 minutes long. There are two parts to the midterm. The first part consists of computational type problems and definition type problems. In the second part, five questions are given, of which you must do three.

You will *not* be allowed to bring in any notes or use the text book, but you may use the standard McMaster calculator. Please bring your **Student Card**.

Material Covered. The midterm will cover the material we discussed in class about Chapters 1-5. Below is a breakdown of what you will need to know from each section. Note that when you are learning definitions, it is good to know an example of that definition, and an example of an object that does not satisfy the definition.

Section 1.1. You won't be tested explicitly on this section, but reading this section is good review since it discusses mathematical proofs.

Section 1.2. Know what we mean by a set, its elements, a proper subset, and the empty set. Know the operations on sets (e.g., union, intersection). Also know how to prove that two sets are equal. Know the Cartesian product, what we mean by relations, the terminology associated to functions (e.g., injective, surjective). Know what an equivalence relation is, and how to prove that a relation is an equivalence relation. Know Theorem 1.25 and and Corollary 1.26. Also know how to do arithmetic modulo n (see Example 1.30).

Section 2.1. Know how to do a proof using induction. Also, know the principle of well-ordering.

Section 2.2. Know Theorem 2.9 and Theorem 2.10. Know the terms: common divisor, greatest common divisor, and relatively prime. Know how to use the Euclidean algorithm to find $\gcd(a, b)$, and how to use this algorithm to find the r and s of Theorem 2.10. Know what we mean by a prime number, and know some of its basic properties, e.g., Lemma 2.13, Theorem 2.14, and Theorem 2.15.

Section 3.1. Know the set $\mathbb{Z}_n$, and its arithmetic properties (e.g., Proposition 3.4). Also, know how to use the symmetry of a geometric figure (like the square) to derive a set of permutations.

Section 3.2. Know the definition of a group. Know terms like binary operation, identity element, inverse, abelian, and non-abelian. Know examples of groups, e.g., $\mathbb{Z}_n$, $U(n)$, $\mathbb{M}_2(\mathbb{R})$, $GL_2(\mathbb{R})$. Know terms like finite group and infinite group. Know the basic properties of a group, e.g., Propositions 3.17-22. Also know Theorem 3.23.

Section 3.3. Know the definition of a subgroup, and some examples of a subgroup. Know how to prove that a given subset is a subgroup (e.g., Proposition 3.30).

Section 4.1. Know the definition of a cyclic group. Know what we mean by the order of an element in a group. Be able to compute the order of an element. Know the properties of cyclic groups, e.g., Theorems 4.9-10. Know Proposition 4.12. Also know Theorem 4.13 and Corollary 4.14.

Section 4.2 Know what the circle group is. Also, know the n-th roots of unity, and the primitive n-th roots of unity. Note you will need to be familiar with the properties of complex numbers.

Section 4.3. You do not need to know this section.

Section 5.1. Know what a symmetric group on n letters is. Know what a permutation group is. Know how to write a permutation using cycle notation. Know what a cycle is, and what it means for cycles to be disjoint. Know Theorem 5.9. Know how to write any permutation as a product of transpositions. Know Theorem 5.15. Know what the alternating group is.

Section 5.2. Know what the n-th dihedral group is. Know Theorem 5.23. You do not need to know about the motions of the cube.

One of the five questions will be taken from among Section 5.3, Exercises 20, 21, 22, or 23.

If you have questions, please feel free to email me. Good luck!