Math 3H03 (Number Theory)
Due: January 24, 2020

Homework Assignment 2

All of the questions from Part A will be graded. One of the questions from Part B will be graded in detail, while the other will be marked for completion. Assignments will be submitted via *Crowdmark*.

Part A. [Short Questions; 4pts]

Exercise 1. Find all solutions to the linear Diophantine equation $172x + 20y = 100$.

Exercise 2. A *continued fraction* is a number of the form

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \cfrac{1}{\ddots}}}}}$$

Any real number α can be turned into a continued fraction using the following procedure to find the a_i's:

- Let $a_0 = \lfloor \alpha \rfloor$, and set $b_0 = \alpha - a_0$.
- For $i \geq 1$, $a_i = \lfloor \frac{1}{b_{i-1}} \rfloor$ and $b_i = \frac{1}{b_{i-1}} - a_i$.
- Stop if $b_i = 0$.

Recall the $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

Find the continued fraction of $\frac{172}{20}$.

Part B. [Proof Questions; 6pts]

Exercise 3. The following problem is over 1200 years and is due to Alcuin of York: One hunderd bushels of grain are distributed among 100 persons so that each man receives 3 bushels of grain, each woman receives 2 bushels of grain, and each child receives $\frac{1}{2}$ bushels of grain. How many men, women, and childern are there? [Hint 1: Use the given information to create a linear Diophantine equation; Hint 2: there are 7 solutions.]

Exercise 4. The *Fibonacci numbers* are defined recursively as follows: $f_0 = f_1 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$. Prove that the continued fraction of $\frac{f_n}{f_{n-1}}$ for $n \geq 1$ has the form

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\cdots + \cfrac{1}{1 + \cfrac{1}{1}}}}}$$

that is, $a_0 = a_1 = \cdots = a_{n-1} = 1$.