Math 3H03 (Number Theory)
Due: January 31, 2020

Homework Assignment 3

All of the questions from Part A will be graded. One of the questions from Part B will be graded in detail, while the other will be marked for completion. Assignments will be submitted via *Crowdmark*.

Part A. [Short Questions; 4pts]

Exercise 1. A *prime triple* is a 3-tuple of numbers of the form $(p, p+2, p+6)$ such that all three numbers are prime. Find a prime triple such that $p \geq 200$.

Exercise 2. Is the Mersenne number M_{19} a Mersenne prime? (You may need a computer to check).

Exercise 3. Find the smallest prime p such that $p-1$ and $p+1$ both have at least three distinct prime divisors.

Part B. [Proof Questions; 6pts]

Exercise 4. Show that there are an infinite number of primes of the form $6n + 5$. [Hint: try to adapt the proof done in class for the case $4n + 3$.]

Exercise 5. Show that for all $n > 4$, $p_n < p_1 + p_2 + \cdots + p_{n-1}$. Here p_n is the n-th prime. [Hint: You may use the following fact without proof: "For any integer n, there is a prime number p such that $n < p < 2n$".]