Math 3H03 (Number Theory)
Due: February 6, 2020

Homework Assignment 4

All of the questions from Part A will be graded. One of the questions from Part B will be graded in detail, while the other will be marked for completion. Assignments will be submitted via *Crowdmark*.

Part A. [Short Questions; 4pts]

Exercise 1. Goldbach's conjectue states that every even number n can be written as $n = a + b$ where a and b are either prime or 1. For example, $8 = 3 + 5$ and $20 = 17 + 3$. Verify Goldbach's conjecture for $n = 2020$.

Exercise 2. Prove that $53^{103} + 103^{53} \equiv 0 \pmod{39}$. [Hint: $14^2 \equiv 1 \pmod{39}$.]

Part B. [Proof Questions; 6pts]

Exercise 3. Let $n = a_t a_{t-1} a_{t-2} \cdots a_3 a_2 a_1 a_0$ be the decimal represenation of n. Show that 7 divides n if and only if

$$7 | (a_2 a_1 a_0) - (a_5 a_4 a_3) + (a_8 a_7 a_6) - \cdots$$

For example, if $n = 24,382,681,421$, then $7|n$ since

$$7 | [(421) - (681) + (382) - 24] \Leftrightarrow 7 | 98.$$

Exercise 4. Suppose that $f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$ is a polynomial such that $f(1) = 2$, $f(2) = 3$, and $f(3) = 5$. Prove that some a_i is not an integer.