MATH 3H03 (NUMBER THEORY)
DUE: MARCH 6, 2020

HOMEWORK ASSIGNMENT 6

All of the questions from Part A will be graded. For Part B, do questions 3 and 4, one of which will be graded completely and the other for completion. Assignments will be submitted via *Crowdmark*.

Part A. [Short Questions; 4pts]

Exercise 1. Consider the congruence equation $x^2 + 3 \equiv 0 \pmod{7^e}$. Use that fact that $x \equiv 2 \pmod 7$ is a solution to $x^2 + 3 \equiv 0 \pmod 7$ and Hensel's Lemma to "lift" this solution to a solution to $x^2 + 3 \equiv 0 \pmod{7^3}$. [Note: you will need to "lift" twice.]

Exercise 2. Find a primitive Pythagorean triplet (a, b, c) where one of a, b, and c is equal to 100.

Part B. [Proof Questions; 6pts]

Exercise 3. Prove that there is no primitive Pythagorean triplet (a, b, c) that contains 6.

Exercise 4. Let (a, b, c) be a Pythagorean triplet. Consider the triplet (d, e, f) where

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} d \\ e \\ f \end{bmatrix}.$$

Prove that (d, e, f) is also a Pythagorean triplet.

Bonus. Show that if we assume that (a, b, c) is a primitive Pythagorean triplet Exercise 4, then (d, e, f) is also a primitive Pythagorean triplet.