Math 4GR3 Final Exam Info Sheet

The purpose of this handout is to help you study by listing the concepts, definitions, and results you will need to know for the final exam.

Exam Information. The final exam will be on **Saturday, April 20, 2024 at 7:30 PM**. The location of the final exam will be announced on MOSAIC. The final exam will be 150 minutes long (2.5 hours).

There are two parts to the final. The first part consists of computational type problems and definition type problems. There are nine questions of this type. You need to do all the questions from this part. In the second part, eight proof questions are given, of which you must do six. The first part of exam is worth 20 points. The second part of the exam is worth 30 points. The exam is out of 50 points.

You will *not* be allowed to bring in any notes or use the text book, but you may use the standard McMaster calculator. Please bring your **Student Card**.

Material Covered. The final exam will be cumulative. For the material covered in Chapters 12-15, see the midterm review sheet. Below is a breakdown of what you will need to know from Chapters 16-18, 21. Note that when you are learning definitions, it is good to know an example of that definition, and an example of an object that does not satisfy the definition. Note that I am assuming you know some of the main results from 3GR3 (e.g., Lagrange's Theorem, First Isomorphism Theorem, the definition of a normal subgroup).

Section 16.1. (The material of this chapter is mostly review of Math 3GR3). Know what we mean by a ring, identity, unit, commutative ring, integral domain, division ring, field. Know what a subring is, and how to verify a subset is a subring (Proposition 16.10).

Section 16.2. Know what we mean by a zero divisor and unit. Know Proposition 16.15, Theorem 16.16. Know what we mean by the characteristic of a ring, and know Theorem 16.19.

Section 16.3. Know what a ring homomorphism is and its properties (Proposition 16.22). Know the definition of an ideal (and how to check if a subset is an ideal). Know what we mean by a trivial ideal and a principal ideal. Know Theorem 16.25. Know how to construct the quotient ring (Theorem 16.29). Know what the kernel of a homomorphism is. Know the various isomorphism theorems on page 253-254.

Section 16.4. Know what a maximal and prime ideal are. Know Theorem 16.35 and Proposition 16.38. Also important is Corollary 16.40.

Section 17.1. Know what a polynomial is and properties like degree and being monic. Know the definition of a polynomial ring. Know Theorem 17.3, Proposition 17.4, and Theorem 17.5.

Section 17.2. Know the division algorithm for polynomials with coefficients in a field. Know what we mean by the greatest common divisor of two polynomials, and Proposition 17.10.

Section 17.3. Know what is meant by an irreducible polynomial. Know Gauss's Lemma (17.14), and its corollary (Cor. 17.15). Know how to use Eisenstein's Criterion to determine if

a polynomial is irreducible over $\mathbb{Q}$. Know some of the properties of ideals in $F[x]$; in particular, know Theorems 17.20 and 17.22.

Section 18.1. Know what is meant by the field of fractions of a domain, and how to construct such a field (Lemma 18.3). Also know Theorem 18.4.

Section 18.2. Know what it means for an element to be irreducible and prime. Know what it means for two elements to be associates. Know the definition of a UFD, PID, and Euclidean Domain. Know Theorem 18.12, Corollary 18.18, Lemma 18.14, Theorem 18.5, Corollary 18.16, Theorem 18.21, Corollary 18.22. For the material on "Factorization on $D[x]$" starting on page 298, you only need to know the statement of Theorem 18.29, but not the proof. The same for the corollaries on page 300. [There is a lot of material in this section!]

Section 21.1. Know what we mean by an extension field and base field. Know Theorem 21.5. Know what we mean for an element to be algebraic or transcendental over a field. Know what an algebraic extension is, and what a simple extension is. Know the difference between an algebraic number and transcendental number. Know Theorems 21.9 and 21.10. Know what we mean by the minimal polynomial of α over F, and what we mean by its degree. Know Proposition 21.12 and Theorem 21.13. Know what a finite extension of degree n over F means, and know Theorems 21.15 and 21.17. Also know Corollary 21.18 and Theorem 21.22. Know what we mean by the algebraic closure of a field, and know examples of algebraically closed fields.

Section 21.2. Know what is meant by a splitting field, and what it means for a polynomial to split. Know Theorem 21.31. You should know the statement of Theorem 21.34, but you don't need to know its proof (same for Lemma 21.32).

Section 21.3. Know what it means for a number to be constructible. Know Theorem 21.37, Lemma 21.41, and Theorem 21.43. Know how the proofs for showing the impossibility of doubling the cube, squaring the circle, and trisecting an angle work.

If you have questions, please feel free to email me. I will post office hours in the week leading up to the final exam. Good luck!