Combinatorial Algebra meets Algebraic Combinatorics
Eighth Annual Meeting

Lakehead University, Thunder Bay, ON
January 21-23, 2011

Schedule and Abstracts

Friday January 21 (Ryan Building 2047 , Lakehead University)

2:00	– 2:30	Registration (Ryan Building 2023)
2:30	– 3:30	*Colloquium: Tony Geramita*
		Coffee Break (Ryan Building 2023)
4:00	– 4:25	*Jaydeep Chipalkatti*
4:35	– 5:25	*Jerzy Weyman*
6:00		Dinner at Outpost – Campus Pub

Saturday January 22 (Valhalla Inn)

8:00		Breakfast (in Conference Room)
8:30	– 9:20	*Francois Bergeron*
9:30	– 9:55	*Christian Stump*
		Coffee Break
10:25	– 10:50	*Steven Sam*
11:00	– 11:25	*Andrew Hoefel*
11:35	– 12:00	*Gregoire Dupont*
		Lunch
1:45	– 2:35	*Jessica Sidman*
2:45	– 3:10	*Ben Babcock*
		Coffee Break
3:35	– 4:00	*Hugh Thomas*
4:10	– 5:00	*Vic Reiner*
7:00		Dinner at Masala Grille

Sunday January 23 (Valhalla Inn)

8:30		Breakfast (in Conference Room)
9:00	– 9:50	*Nantel Bergeron*
10:00	– 10:25	*Vivien Ripoll*
10:35	– 11:00	*Luis Serrano*

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Abstracts

Title: REVISITING THE SPREADING AND COVERING NUMBERS

Speaker: Ben Babcock (Lakehead)

Abstract: Let R be a polynomial ring in n variables and S denote the monomials of degree d in R. Take any subset of S and multiply it by the indeterminates of R. If the resulting monomials are distinct, we say they are "spread out" among the monomials of degree $d+1$. The spreading number, $\alpha_n(d)$, is the cardinality of the largest subset of S that satisfies this property. The covering number, $\rho_n(d+1)$, is the cardinality of the smallest subset of S that generates all the monomials of degree $d+1$.

This talk is an overview of research done during a summer NSERC USRA. We examine two methods of computing $\alpha_n(d)$: constructing a simplicial complex and computing its dimension, and using the symmetry of a graph. Additionally, we present an algorithm for computing upper bounds of $\rho_n(d+1)$ that improves upon the explicit formula for general n, d.

Title: INVERSE SYSTEMS FOR DIAGONAL QUASI-INVARIANTS OF COMPLEX REFLECTION GROUPS

Speaker: François Bergeron (UQÀM)

Abstract: We will present recent results and questions concerning the inverse systems associated to diagonal quasi-invariants (in many sets of variables) for finite complex reflection groups obtained as generalized symmetric groups. In particular, we will see that their Hilbert series have an especially nice form.

Title: ON AN LR RULE FOR THE SCHUBERT VARIETIES AS GRASSMANIAN MODULE

Speaker: Nantel Bergeron (York)

Abstract: A positive (no cancellation) rule for the multiplication of Schubert polynomials is still out of hand. With F. Sottile (in 1999), we hoped that the multiplication of a Schubert polynomial by a Schur function could be simpler to resolve and we gave a symmetric function that encode these numbers when expanded in term of Schur functions. This remain unsolved all these years... Recently (2010), with S. Assaf and F. Sottile, we have a much better understanding of the complexity of the problem. We give a combinatorial proof of the positivity of the numbers, provide a rule for many of them (including all the presently known rules) and we give a combinatorial explanation of why the other cases are so hard.

Title: INVERSE SYSTEMS OF TRIVARIATE SYMMETRIC POLYNOMIALS

Speaker: Jaydeep Chipalkatti (Manitoba)

Abstract: Consider the two polynomial rings $R = Q[x_1, x_2, x_3]$, and $S = Q[y_1, y_2, y_3]$. We can define the structure of an R-module on S by identifying x_i with the differential operator $\frac{\partial}{\partial y_i}$. If $F \in S$ is a homogeneous polynomial of order d, then the ideal $I_F = \text{ann}(I) \subseteq R$ is called the inverse system associated to F . It is a Gorenstein ideal of codimension three, and as such admits a self-dual minimal free resolution

$$0 \leftarrow R/I_F \leftarrow R \xleftarrow{q_0} M \xleftarrow{q_1} M^\vee \oplus R(-d-3) \xleftarrow{q_3} R(-d-3) \leftarrow 0,$$

according to the Buchsbaum-Eisenbud structure theorem. In particular, I_F is generated by the Pfaffians associated to the (skew-symmetric) morphism q_1 .

Now further suppose that F is symmetric in the variables y_i . Then each Betti module in the resolution is a representation of the symmetric group $\mathfrak{S}_3$, and all the coboundary morphisms are equivariant. I will present some results and conjectures about this resolution when F is either the Schur polynomial associated to a partition of d, or a general symmetric d-ic.

Title: A REPRESENTATION-THEORETICAL APPROACH TO CLUSTER BASES IN CLUSTER ALGEBRAS

Speaker: Gregoire Dupont (Sherbrooke)

Abstract: This is a report on joint works with G. Cerulli (U. di Roma), F. Esposito (U. di Padova) and H. Thomas (U. of New Brunswick). A cluster basis in a cluster algebra is a linear basis containing the set of cluster monomials. In this talk, I will present various constructions of cluster bases in cluster algebras using representation theory of quivers. I will in particular explain the construction of a representation-theoretical context for the combinatorial constructions of canonically positive bases (or atomic bases) considered by Sherman- Zelevinsky and Cerull.

Title: SUMS OF SQUARES: EVOLUTION OF AN IDEA

Speaker: Speaker: Anthony Vito Geramita (Queen's and Genoa)

Abstract: Beginning with Fermat's characterization of primes which are the sum of two squares and Lagrange's 4-squares Theorem (with Gauss' addendum) I will discuss how these two theorems naturally lead to the famous Waring Problems for integers. With that as a backdrop, we will see that there is an analogous formulation of Waring's Problems for homogeneous polynomials.

These Polynomial Waring Problems have a beautiful geometric reformulation and once that reformulation is apparent, other similar problems become very apparent and are the target of much current research. Problems, for example, which generalize the notion of rank for matrices.

Moreover, these problems have important applications in Phylogenetics and Communication Theory as well as in Statistics.

Title: GOTZMANN SQUAREFREE MONOMIAL IDEALS

Speaker: Andrew Hoefel (Dalhousie)

Abstract: Let $S = k[x_1, \ldots, x_n]$ be the polynomial ring and $R = S/(x_1^2, \ldots, x_n^2)$ be the Kruskal-Katona ring. A homogeneous ideal $I \subset S$ (or R) is called Gotzmann if each graded component has the smallest possible Hilbert function given its number of generators. Gotzmann squarefree monomial ideals I of S can be classified using properties of IR. Though the problem of classifying Gotzmann monomial ideals of R seems more difficult, certain decomposition and reconstruction results can be given. Gotzmann ideals have a number of nice algebraic properties and Gotzmann monomial ideals of R arise in interesting combinatorial problems.

Title: P-PARTIONS REVISITED

Speaker: Vic Reiner (Minnesota)

Abstract: (This is joint work with Valentin Feray) Counting the linear extensions of a general partially ordered set (poset) is hard. We'll explain a new product formula which works for a certain class of posets, generalizing a formula for forest posets due to Knuth, and its q-generalization by Bjorner and Wachs.

We'll also explain how this formula arises naturally when one re-examines Stanley's P-partitions from the perspective of convex cones and their affine semigroup rings.

Title: FACTORISATIONS OF A COXETER ELEMENT AND DISCRIMINANT OF A REFLECTION GROUP

Speaker: Vivien Ripoll (UQAM)

Abstract: When W is a finite reflection group, the noncrossing partition lattice NCP_W of type W is a very rich combinatorial object, extending the notion of noncrossing partitions of an n-gon. A formula (for which the only known proofs are case-by-case) expresses the number of multichains of a given length in NCP_W as a generalized Fuss-Catalan number, depending on the invariant degrees of W. We explain how to understand some specifications of this formula in a case-free way, using an interpretation of the chains of NCP_W as fibers of a "Lyashko-Looijenga covering". This covering is constructed from the geometry of the discriminant hypersurface of W. We deduce new enumeration formulas for certain factorisations of a Coxeter element of W.

Title: SOME RECENT DEVELOPMENTS IN BOIJ-SODERBERG THEORY

Speaker: Steven Sam (MIT)

Abstract: Boij-Soderberg theory provides a radically new approach to thinking about minimal free resolutions over the polynomial ring by replacing the arithmetic of modules with the combinatorics of Betti tables. Two questions: what information is lost via this process? and can we work over more general rings? I'll give an introduction to Boij-Soderberg theory and describe some recent progress on answering these two questions.

Title: THE SHIFTED PLACTIC MONOID

Speaker: Luis Serrano (UQAM)

Abstract: We introduce a shifted analog of the plactic monoid of Lascoux and Schutzenberger, the shifted plactic monoid. It can be defined in two different ways: via the shifted Knuth relations, or using Haiman's mixed insertion. Applications include: a new combinatorial derivation (and a new version of) the shifted Littlewood-Richardson Rule; similar results for the coefficients in the Schur expansion of a Schur P-function; a shifted counterpart of the Lascoux-Schutzenberger theory of noncommutative Schur functions in plactic variables; a characterization of shifted tableau words; and more. This is based on http://arxiv.org/abs/0811.2057.

Title: SYZYGIES OF GRAPH CURVES AND THEIR SECANT VARIETIES

Speaker: Jessica Sidman (Mt. Holyoke)

Abstract: Let G be a simple connected graph with d vertices and n edges. The graph curve associated to G is a curve with only nodes as singularities constructed by taking one copy of P^1 for each vertex and requiring that two lines intersect if the corresponding vertices are joined by an edge in G. Graph curves associated to trivalent graphs have been studied by Bayer and Eisenbud as well as Ciliberto and Miranda. We study embeddings of graph curves in projective space for which d is at least $2g+1$. We will discuss how the syzygies of graph curves and their secant varieties compare to what we know for smooth curves.

This is joint work with Greg Burnham, Zvi Rosen, and Peter Vermeire.

Title: MAXIMAL FILLINGS OF MOON POLYOMINOES, SIMPLICIAL COMPLEXES, AND SCHUBERT POLYNOMIALS

Speaker: Christian Stump (UQAM)

Abstract: We exhibit a canonical connection between maximal (0,1)-fillings of a moon polyomino avoiding north-east chains of a given length and reduced pipe dreams of a certain permutation. Following this approach we show that the simplicial complex of such maximal fillings is a vertex-decomposable, and thus shellable, sphere. In particular, this implies a positivity result for Schubert polynomials.

Title: CYCLIC POLYTOPES AND HIGHER DIMENSIONAL ANALOGUES OF CLUSTER ALGEBRA COMBINATORICS

Speaker: Hugh Thomas (UNB)

Abstract: The simplest cluster algebras are those associated to triangulations of a polygon. I will discuss a higher-dimensional analogue of the combinatorics of these cluster algebras, in which the polygon has been replaced by an even-dimensional cyclic polytope. I will also mention the links to representation theory (a generalization of the cluster category), and to the geometry of laminations. This is joint work with Steffen Oppermann, and is mostly based on arXiv:1001.5437.

Title: ORBIT CLOSURES IN REPRESENTATION WITH FINITELY MANY ORBITS

Speaker: Jerzy Weyman (Northeastern)

Abstract: Let L be a simple Lie algebra, and let α be a simple root in L. The root α denotes a $\mathbb{Z}$-grading on L. We are interested in the action of the adjoint group G_0 of a Lie algebra L_0 on the space L_1. Such representations are closely related to irreducible representations of semi-simple Lie groups with infinitely many orbits. It is well known that the action of $G_0 x C^*$ on L_1 has infinitely many orbits. By using geometric invariant theory we calculate Hilbert polynomials of (normalizations) of orbit closures. In many cases we can deduce normality, CohenMacaulay and Gorenstein properties of the orbit closures. Interesting examples will be discussed. The talk is based on a joint work with Witold Kraskiewicz.

Participants

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We would like to thank the following organizations for making this conference possible:

- The Fields Institute for Research in Mathematical Sciences,
- AARMS: Atlantic Association for Research in the Mathematical Sciences,
- NSERC: Natural Sciences and Engineering Research Council of Canada,
- Lakehead University: Mathematical Sciences, Office of the Dean of Science and Environmental Sciences, and Office of the VP (Academic).