Math 2X03 - Homework 4

Due: May 31, 2018 (by 10:00 pm)

(The following problems are from the textbook.)

1. (§16.3 #18)
   (a) Find a function f such that $\vec{F} = \nabla f$, where $\vec{F}(x, y, z) = (\sin y)\vec{i} + (x \cos y + \cos z)\vec{j} - (y \sin z)\vec{k}$.
   (b) Use part (a) to evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the curve C given by $\vec{r}(t) = (\sin t)\vec{i} + t\vec{j} + 2t\vec{k}, 0 \leq t \leq \pi/2$.
2. (§16.4 #4) Evaluate the line integral $\oint_C x^2y^2dx + xydy$, where C consists of the arc of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$ and the line segments from $(1, 1)$ to $(0, 1)$ and from $(0, 1)$ to $(0, 0)$, by two method: (a) directly and (b) using Green's Theorem.
3. (§16.4 #8) Use Green's Theorem to evaluate the line integral $\int_C y^4dx + 2xy^3dy$, where C is the ellipse $x^2 + 2y^2 = 2$ with counter-clockwise direction.
4. Determine whether or not the vector field is conservative. If it is conservative, find a function f such that $\vec{F} = \nabla f$.
   (a) (§16.5 #14) $\vec{F} = (xyz^4)\vec{i} + (x^2z^4)\vec{j} + (4x^2yz^3)\vec{k}$.
   (b) (§16.5 #18) $\vec{F} = (e^x \sin(yz))\vec{i} + (ze^x \cos(yz))\vec{j} + (ye^x \cos(yz))\vec{k}$.