Complex Analysis -Assignment Two

due Tuesday, Jan 30

1. Show that

$$\int_{-\infty}^{\infty} \left(\frac{\cos x}{x^2 + a^2} \right) dx \;=\; \left(\frac{\pi}{a} \right) e^{-a}$$

for all real number $a > 0$.

2. Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{dx}{1 + x^4}.$$

3. Show that

$$\left(\frac{\pi}{2} \right) (1 + 2\pi|\xi|)\, e^{-2\pi|\xi|} \;=\; \int_{-\infty}^{\infty} e^{2\pi \mathbf{i} x \xi} \left(\frac{1}{1 + x^2} \right)^2 dx$$

for all $\xi \in \mathbb{R}$.

4. Show that

$$\int_{-\infty}^{\infty} \left(\frac{1}{1 + x^2} \right)^{n+1} dx \;=\; \left(\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} \right) \pi$$

for every $n \in \mathbb{N}$.