Complex Analysis -Assignment Five

due Friday, March 9

1. Show that if $f \in \mathfrak{S}_a$ for some $a > 0$, then for any positive integer n, one has $f^{(n)} \in \mathfrak{S}_b$ whenever $0 \leq b < a$.

2. Prove that

$$\frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{a}{a^2 + n^2} = \sum_{n=-\infty}^{\infty} e^{-2\pi a|n|}$$

whenever $a > 0$.

3. Let τ to be fixed with $\mathbf{Im}(\tau) > 0$. By applying Poisson summation formula to

$$f(z) = (\tau + z)^{-k}$$

where k is an integer ≥ 2, obtain the identity

$$\sum_{n=-\infty}^{\infty} \left(\frac{1}{\tau + n}\right)^k = \frac{(-2\pi \mathbf{i})^k}{(k-1)!} \sum_{m=1}^{\infty} m^{k-1} e^{2\pi \mathbf{i} m\tau}.$$

Hint: Use Residue formula to show that

$$\widehat{f}(\xi) = 0, \text{ for } \xi < 0 \qquad \text{and} \qquad \widehat{f}(\xi) = \frac{(-2\pi \mathbf{i})^k}{(k-1)!} \xi^{k-1} e^{2\pi \mathbf{i} \xi \tau}, \text{ for } \xi > 0.$$