Complex Analysis -Assignment One

due Tuesday, Jan 23

1. Show that

$$\int_0^\infty \left(\frac{\sin x}{x}\right) dx \;=\; \frac{\pi}{2}.$$

Hint: Use semicircle contour.

2. Evaluate the integrals

$$\int_0^\infty e^{-ax}\cos(bx)dx, \qquad \int_0^\infty e^{-ax}\sin(bx)dx$$

for $a > 0$.

Hint: Integration along a sector contour with appropriate angle.

3. Prove that for every $\xi \in \mathbb{C}$, we have

$$e^{-\pi\xi^2} \;=\; \int_{-\infty}^\infty e^{-\pi x^2} e^{2\pi \mathbf{i} x\xi} dx.$$

4. Let $a, b \in \mathbb{C}$ such that $|a| < |b|$. SHow that if $|a| < r < |b|$ and $\mathfrak{C}$ is the positively oriented circle centered at the origin with radius r, then

$$\int_{\mathfrak{C}} \frac{dz}{(z-a)(z-b)} \;=\; \frac{2\pi\mathbf{i}}{a-b}.$$