Complex Analysis -Assignment Three

due Friday, Feb 9

1. Let $a, b \in \mathbb{R}$ and $|a| > |b|$. Show that

$$\int_0^{2\pi} \frac{d\vartheta}{a + b\cos\vartheta} \;=\; \frac{2\pi}{\sqrt{a^2 - b^2}}.$$

2. Show that if $a > 0$, then

$$\int_0^{\infty} \frac{\log x}{x^2 + a^2} dx \;=\; \frac{\pi}{2a} \log a.$$

for $a > 0$.

Hint: Use the contour of simicircles.

3. Prove that

$$\int_0^{2\pi} \log\left|1 - e^{i\vartheta}\right| d\vartheta \;=\; 0.$$

Hint: Prove that if $|a| < 1$,

$$\int_0^{2\pi} \log\left|1 - ae^{i\vartheta}\right| d\vartheta \;=\; 0$$

Then, extend the above result to $|a| \leq 1$.